MONTHLY NOTICES

OF THE

ROYAL ASTRONOMICAL SOCIETY.

VOL. LXXXIII. MAY 11, 1923. No. 7

Dr. J. L. E. DREYER, PRESIDENT, in the Chair.

Frederic Douglas Addyman, L.C.C. School of Engineering and Navigation, High Street, Poplar, London, E. 14;
Harold Reynolds Kingston, M.A., Ph.D., Professor of Mathematics and Astronomy at Western University, London, Ontario, Canada; and
Arthur Robert Wesley, 9 Devonport Street, Lancaster Gate, London, W. 2,

were balloted for and duly elected Fellows of the Society.

The following candidates were proposed for election as Fellows of the Society, the names of the proposers from personal knowledge being appended:—

Albert Clapham, 15 Lonsdale Road, Blackpool (proposed by T. P. Witter);
Frederick H. Clark, c/o Mineral Separation Ltd., 62 London Wall, London, E.C. 2 (proposed by R. T. A. Innes);
Rev. J. Silas Evans, B.A., The Vicarage, Llanrhaiadr, Oswestry (proposed by W. S. Warren);
William David Evans, M.A., 68 Argyle Road, West Ealing, London, W. 13 (proposed by A. Berry);
Stanislas Kalinowski, Director of the Magnetic Observatory of Swider, rue Gorna 10, Warsaw, Poland (proposed by J. Krassowski);
Philip Sidney Langton, F.R.Met.Soc., Assistant in the Meteorological Service of Canada, 1016 Verrinder Avenue, Victoria, B.C., Canada (proposed by J. S. Plaskett);
Robert Peebles, 12 Wood Street, London, E.C. 2 (proposed by F. Robbins);

John William Perry, The Research Laboratory, Adam Hilger Ltd., and 24 Dalmeny Avenue, London, N. 7 (proposed by F. Robbins); and

Archibald C. T. Stevens, c/o Messrs. Wilson & Sons Ltd., 69 Bishopsgate, London, E.C. 2, and Alma House, Freshwater, Isle of Wight (proposed by R. T. A. Innes).

Sixty-seven presents were announced as having been received since the last meeting, including, amongst others:—

Algiers Observatory, Astrographic Chart, Zones $+1^\circ$ and $+3^\circ$ (30 charts);

Bordeaux Observatory, Astrographic Chart, Zone $+16^\circ$ (2 charts);

R. Jarry-Desloges, Observations des Surfaces Planétaires, Années 1915, 1916;

Toulouse Observatory, Astrographic Chart, Zones $+9^\circ$, $+7^\circ$, and $+5^\circ$ (29 charts);

R. Woolsey, The Celestial Companion (1802) (presented by Mrs. Hodson);

Transparency of the Moon, by Warren de la Rue (presented by Mrs. Grove-Hills); and

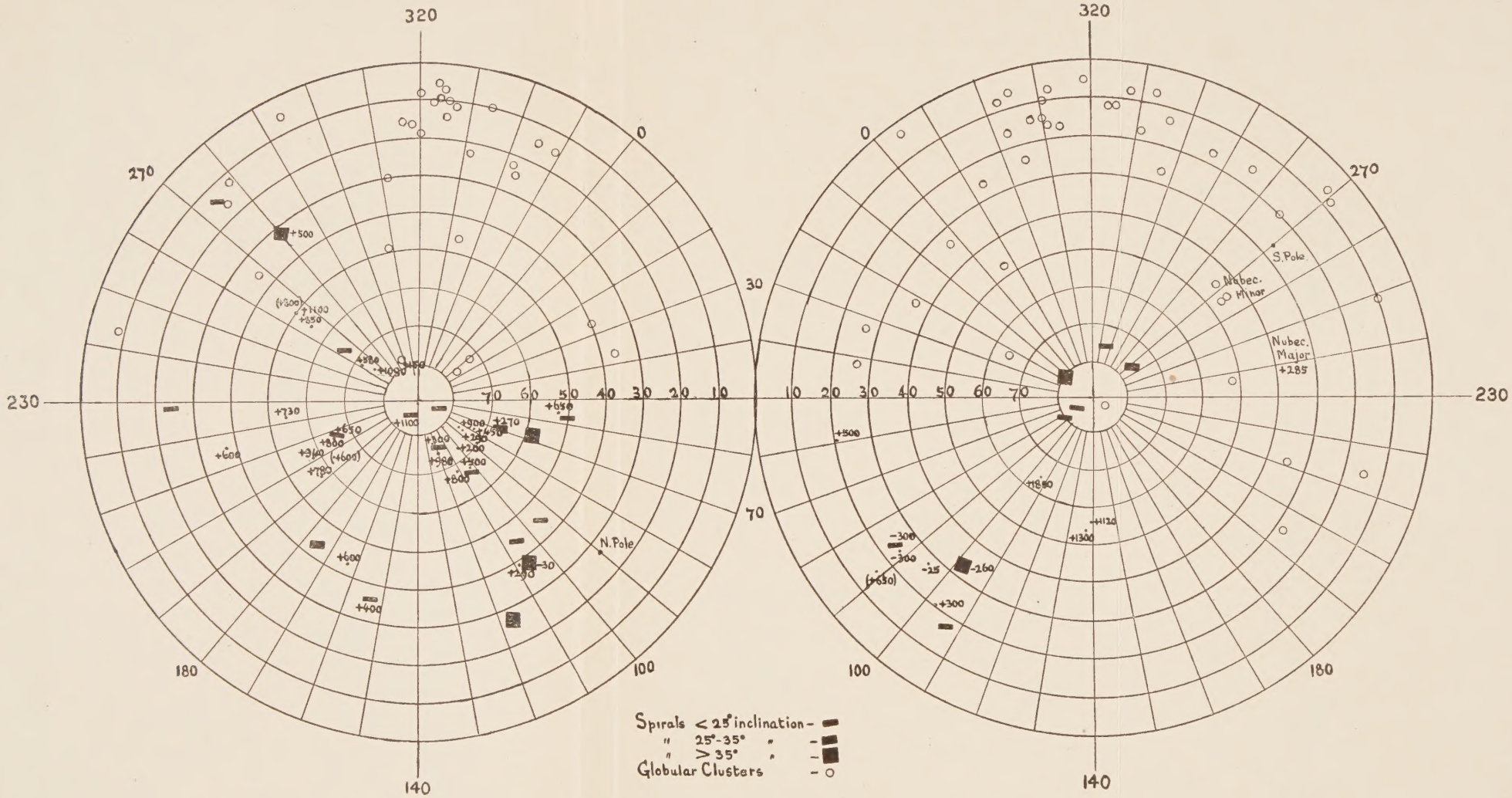
A Globe of the Moon, by J. Russell (1797) (presented by Miss Bacon).

Note on Professor Lindemann's Paper on the Constitution of the Spiral Nebulæ. By J. H. Reynolds. (Plate 14.)

The hypothesis suggested by Professor Lindemann that the spiral nebulæ originated in dust clouds expelled from the Galactic plane by radiation pressure is interesting and important. In the opinion of many astronomers the external galaxy theory of the spirals has failed signally to explain certain features which have been established in recent years, and a fresh standpoint such as he suggests is very desirable in order to deal with the problem of their real nature.

There are other facts besides those mentioned by Dr. Lindemann which seem to favour his hypothesis. The principal of these is the galactic distribution. A full discussion as to the distribution of the spiral nebulæ down to $2'$ diameter will be found in *M.N.*, 83, 147, and if we follow Shapley in presuming that the solar system is not almost centrally situated in the galaxy, but considerably eccentric in the proportion of 3 to 1, the apex lying in galactic longitude 320°, we should expect the dust clouds to be swept away from the region of greatest radiation pressure in a diagonal direction.* If we take the number of stars in unit volume in globular clusters as of the order

* According to H. N. Russell, the radiation pressure of the B-type stars would be about 100 times that of gravitational attraction for particles of the optimum diameter. On this moderate assumption the radial velocities of dust clouds of these particles might easily attain a velocity of over 1000 km/sec.



GALACTIC DISTRIBUTION OF THE KNOWN RADIAL VELOCITIES OF THE SPIRAL NEBULAE.

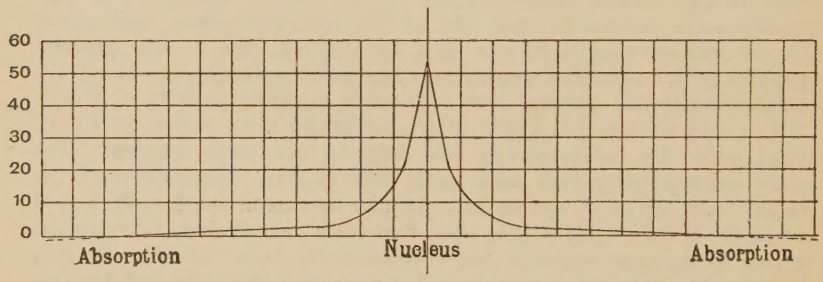
of 10^4 times greater than it is in the neighbourhood of the sun, then undoubtedly the greatest radiation pressure would lie in and near the apex λ320°, as this is not only the region of greatest star density in the galaxy, but also the pole of the hemisphere containing the globular clusters, which lie for the most part between galactic longitudes 230° and 50° (through 360°). The great proportion of globular clusters are within 30° of the galactic equator, and we should expect the spirals in the neighbourhood closest to the apex to be swept away to the greatest distance, and therefore to subtend small angular diameters. We should also expect those of greatest positive radial velocities to lie nearest to that apex in the hemisphere λ230° to 50° (through 360°), and those of smallest positive radial velocities and possibly of small negative velocities in the other hemisphere. This seems to be actually the case in the North Galactic Hemisphere. There is no doubt that the great majority of the large spiral nebulæ are concentrated in the hemisphere λ50° to 230°, while smaller spirals in the N.G.H. are found to preponderate in regions towards and in the semihemisphere represented by the galactic longitudes 230° and 50° (through 360°). The greatest positive radial velocities appear near the north galactic pole along the median longitudes 90° and 270°. The only negative velocity is N.G.C. 3031 (−25 km./sec.) in λ108°, β39°, and in the same region is N.G.C. 3034 with the comparatively low velocity of +250 km./sec. The plate accompanying this note gives the galactic distribution of the radial velocities of 37 spirals and globular nebulæ in the N.G.H. and of 10 in the S.G.H.* It will be seen that some remarkably low velocities appear in the same region as the velocities over 1000 km./sec. It may be that these low radial velocity nebulæ are near the normal of the line of sight with the direction of motion, but in that case we should expect them to be of large angular diameter compared with spirals of great radial velocity, which is not the case. Small negative radial velocities may be explained by the spirals not having yet reached the normal of the line of sight to the direction of motion. In the South Galactic Hemisphere a group of four negative velocities appear in the Andromeda region, including the Andromeda Nebula itself and M. 33, while a group between them and the pole contains three spirals of large positive radial velocities. In the S.G.H. the smaller spirals are more widespread than in the N.G.H., but so are the globular clusters.

As will be seen from the diagrams accompanying the paper on the "Galactic Distribution of Spiral Nebulæ" in *M.N.*, 83, No. 3, the spirals and the globular clusters are, broadly speaking, mutually exclusive, and the quarter of the sphere bounded by galactic longitudes 320° and 50° contains a preponderance of globular clusters but very few spirals. Professor Lindemann points out that when the dust clouds have become condensed into discrete nuclei, it is probable that the gravitational attraction of the galaxy would counterbalance the radiation pressure, and the spiral would then approach the galaxy, unless the initial velocity of recession were very large. So far, therefore, as the distribu-

* The nebulæ in this diagram are taken from a list compiled by Dr. Slipher for Professor Eddington, who has kindly furnished the writer with a copy. Doubtful velocities are inserted in a parenthesis.

tion of the spirals and their radial motions are concerned, Dr. Lindemann's hypothesis is not inconsistent with the known facts.

We now come to the other part of his hypothesis which supposes the illumination of these objects to be affected by reflected starlight from the galaxy. Against this, it seems to the writer that cogent arguments can be adduced. In the first place the light distribution of the uncondensed spirals is of a nature which indicates intrinsic illumination in the spirals themselves. The heaping up of light in the nucleus in spirals of this type is usually very great, the light curve approximating in the case of the Andromeda Nebula to the inverse square (x^2y = constant) and in others to the curve xy = constant, while in the periphery illumination ceases altogether and there appears a dark area of absorbing matter, which is often very conspicuous in photographs of spirals having their planes inclined at a small angle to the line of sight. We may take the diagram given below as typical of the uncondensed spirals.



Generalised Curve of the Light Distribution in the uncondensed Spiral Nebulæ. Based on Photometric Measures.

The presence of the dark band of absorbing matter in edge-on spirals is very hard to explain on any other hypothesis than nuclear illumination.

Dr. Lindemann also takes the general stellar illumination in the neighbourhood of the sun as equal to a fifth magnitude star per square degree. As the spirals and globular nebulæ have sensible angular diameters, the illumination should on the hypothesis be something of this order for nebulæ lying farthest away from the apex $\lambda 270°$, where it would presumably be greater. As a matter of fact, the brightest known spiral and globular nebulæ, which lie farthest from the apex, concentrate something like the light of a seventh magnitude star in a square minute of arc, and concentrations of the light of a ninth magnitude star per square minute of arc are quite numerous, which seems to be altogether inconsistent with this part of the hypothesis. It was suggested in 1913 by the writer in a study of the light distribution of the Andromeda Nebula that there might be a central source of illumination in the nebula itself. If we are to take the colour distribution in the spirals found by Seares and the writer as a real effect and not a photographic one, then we must agree with Professor Lindemann in assuming that the spirals consist of dust concentrations. Some of the spirals have every appearance of this, notably spirals such as N.G.C. 4826. If therefore the reflection hypothesis is to be adopted

on other grounds, there is far more evidence for reflection from the bright nucleus itself than from the distant galaxy. A further difficulty is the type of spectrum given by the majority of spirals, which is usually of the F and G stellar types. Such spectra indicate a temperature of the order of 6000° C., so that a hot mass or masses of this temperature must be postulated as responsible for the light, and this is one of the grounds given by Professor Lindemann for the supposed reflection from the galaxy. At the same time some spirals yield exceptional types of spectra, including bright hydrogen and nebulium lines, which are consistent only with a much higher temperature (*e.g.* N.G.C. 1068). This again seems certainly to indicate an intrinsic illumination in the nebula itself. While the hypothesis is very valuable as affording a reasonable explanation of the origin of the spirals, and of their radial velocities, it does not seem to give an adequate explanation of their illumination. One may indeed question whether it is necessary to assume an actual stellar state for the production of continuous spectra with absorption lines. The existence of continuous spectra in gaseous emission nebulæ of the planetary class and the continuous spectrum and absorption lines given by exploded wires seems to indicate that different physical conditions may produce either emission or absorption types of spectra.

If the light of the outer regions is reflected from particles of dust, we should have expected some evidence of polarisation. Some experiments carried out by the writer with polarising apparatus in 1912 met with no success owing to instrumental difficulties, but this line of work has again been attacked at the Lick Observatory under Director Campbell by Meyer. No definite evidence was found as to polarisation in the spirals according to *L.O. Bulletin*, No. 328, but the exposures were hardly long enough for such work. The spirals most likely to show polarisation would be those similar to N.G.C. 4826 previously mentioned, where the major and minor diameters are in the relation of 2 to 1, and the greatest polarisation effect would be expected if the plane of the Nicol were placed so as to coincide with the direction of the major and minor axes for the two exposures. It would seem worth while to continue this work on suitable spirals with longer exposures. Dr. Jeans has treated the question of spiral formation theoretically as a problem of development from a gaseous nucleus, but he has recognised the possibility of dust in his *Stellar Dynamics, and Problems of Cosmogony.*

The Stellar Magnitude Scales of the Astrographic Catalogue. Seventeenth Note: The San Fernando Magnitudes for −3° and −5°, and the Paris Magnitudes for +22°, +23°, and +24°. By H. H. Turner, D.Sc., F.R.S., Savilian Professor.

1. In this matter of the Astrographic Catalogue, which was a good deal in arrear even before the War, it is always a pleasure to welcome progress in new directions. Sr. Herrero has published three volumes of measures from the San Fernando Observatory (−3° to −9° immediately

south of the four French zones which extend from $+24^\circ$ to -2°), and Sr. Gallo has published a volume of tables and a volume of measures in the Tacubaya Zone (-10° to -16°) which lies between San Fernando and Hyderabad. Thus in spite of some present difficulties, we may soon hope to have the portion from $+31^\circ$ to -51° (Oxford, Paris, Bordeaux, Toulouse, Algiers, San Fernando, Tacubaya, Hyderabad, Cordoba, Perth, Cape) complete, as well as the northern cap $+90^\circ$ to $+55^\circ$ (Greenwich and the Vatican). There remains the difficult gap $+54^\circ$ to $+32^\circ$ (Catania, Helsingfors, Potsdam), and the really serious vacancy at the southern cap -52° to -90° (Sydney and Melbourne) which seems to yield to no treatment.

2. Counts of two of the three zones published by San Fernando, have kindly been supplied by Sr. Herrero as in Tables I. and II., and have been analysed by the uniform procedure adopted in recent papers.

TABLE I.

Counts of Zone -3° (San Fernando) for each hour.
Numbers of plates, 7, 8, 7, 8, etc.

h	]8·5.	8·5.	9·0.	9·5.	10·0.	10·5.	11·0	11·5	[11·5.	All.
0	44	40	77	130	194	246	204	140	22	1097
1	58	50	95	167	221	277	233	121	18	1240
2	50	46	97	128	187	284	209	71	0	1072
3	77	55	98	130	246	376	371	296	6	1655
4	81	66	111	156	255	384	415	429	39	1936
5	97	98	160	207	284	376	407	692	211	2532
6	106	135	276	399	801	1295	1151	1370	324	5857
7	124	182	332	544	1221	1574	1293	965	55	6290
8	90	105	213	382	647	920	695	317	8	3377
9	75	100	214	308	541	508	280	83	1	2110
10	55	58	118	201	288	251	200	113	5	1289
11	54	53	121	242	326	373	180	23	6	1378
12	45	36	83	175	242	297	171	47	0	1096
13	43	69	114	243	318	373	251	129	57	1597
14	39	43	89	181	237	325	236	103	38	1291
15	62	42	82	194	306	405	256	62	2	1411
16	66	58	107	286	342	483	336	98	45	1821
17	84	68	127	239	409	697	449	94	31	2198
18	96	64	111	199	272	339	437	309	136	1963
19	85	85	209	431	623	939	838	646	96	3952
20	63	85	142	359	616	963	1340	979	292	4839
21	58	76	111	223	383	547	526	452	390	2766
22	45	51	100	165	312	447	453	191	76	1840
23	42	46	107	168	266	412	383	73	13	1510
Sum	1639	1711	3294	5857	9537	13091	11314	7803	1871	56117

TABLE II.

Counts for Zone $-5°$. *Numbers of Plates* 7, 8, 7, 8, *etc.*

h	]8·5.	8·5.	9·0.	9·5.	10·0.	10·5.	11·0.	11·5.	[11·5.	All
0	36	40	46	103	164	252	167	103	12	923
1	46	38	72	128	238	262	153	33	3	973
2	35	32	69	144	233	245	117	36	4	915
3	55	50	88	192	271	286	223	112	8	1285
4	62	72	98	158	281	373	207	117	22	1390
5	84	83	132	209	326	497	421	331	89	2172
6	84	93	217	286	417	730	795	1116	603	4341
7	83	147	264	506	862	1204	1173	1454	860	6553
8	59	92	211	306	390	580	445	463	143	2689
9	44	89	127	215	278	270	209	157	36	1425
10	27	42	87	164	175	169	131	75	58	928
11	43	55	78	173	209	243	222	162	92	1277
12	37	40	101	141	236	248	196	151	37	1187
13	52	44	96	195	266	342	217	174	37	1423
14	39	50	89	189	223	244	163	100	39	1136
15	50	53	129	247	369	362	277	207	49	1743
16	38	47	97	234	302	330	205	99	23	1375
17	57	26	81	163	205	248	235	107	24	1146
18	67	71	106	182	310	425	425	699	394	2679
19	73	99	134	282	515	550	450	492	239	2834
20	60	53	114	261	528	628	636	686	319	3285
21	56	56	108	212	423	477	381	328	79	2120
22	33	45	91	151	244	250	229	184	40	1267
23	37	50	79	123	253	266	212	160	33	1213
Sum	1257	1467	2714	4964	7718	9481	7889	7546	3243	46279

The totals for each hour do not accord with the sum of the totals for each plate, because stars on the overlap of two plates were assigned to one plate only and not to both. This reduces the numbers a little, but on consideration it was decided that the difference from other counts was not serious enough to merit special corrections. The symbols] and [are used for "brighter than" and "fainter than" as suggested in *Harvard Bull.*, 781.

The analysis follows the lines of *M.N.*, 78, 54, adopted in subsequent papers. Cumulative totals were formed up to and including 8·5, 9·5, 10·5, and All. Of these the logarithms were taken, and after subtracting log 4 (the number of square degrees per plate) and log 7 or 8 (number

of plates in an hourly group) are considered to be values of $\log N_\gamma$ at Galactic Latitude γ, from which $\log N_0$ is deduced by the formula

$$\log N_0 = (1 + \cdot 0025\gamma) \log N_\gamma + K_\gamma.$$

The resulting mean values of $\log N_0$ are shown in Table IV.: in Table III. the differences from these are given, providing the material for harmonic analysis in Table IV. At the foot of each part of Table IV. the sum of the first two lines is subtracted from the sum of the second, showing the changes for 1·77 and 1·74 magnitudes. The change per magnitude is found, in the lowest lines, by simple division by these numbers.

TABLE III.

San Fernando Residuals after Correction for Galactic Latitude.

(The unit is 0·01 in $\log N_0$.)

	Zone −3°.					Zone −5°.				
	Gal. Lat.	8·5.	9·5.	10·5.	All.	Gal. Lat.	8·5.	9·5.	10·5.	All.
h	°					°				
0	−64·5	+ 1	0	0	+ 1	−66·5	+ 5	− 3	+ 6	+ 3
1	−62	+ 6	+ 5	+ 1	− 2	−64	+ 3	+ 1	+ 7	− 2
2	−54	+ 4	+ 1	− 1	− 7	−55·5	− 3	+ 5	+ 9	− 3
3	−43	+ 8	− 7	− 6	− 1	−44	+ 7	+ 7	+ 4	− 1
4	−30·5	+ 7	− 7	−11	− 2	−31·5	+14	+ 1	+ 2	− 6
5	−17·5	+ 6	−19	−30	−18	−18·5	+11	0	− 3	− 1
6	− 4	− 3	− 7	− 2	+ 7	− 5	− 7	−11	−13	+ 5
7	+ 9·5	+ 6	+ 3	+10	+10	+ 8·5	+ 1	+ 3	+ 8	+21
8	+22·5	+11	+12	+15	+12	+21·5	+ 6	+10	+ 7	+ 9
9	+35	+ 4	+17	+13	+ 2	+33·5	+ 9	+ 7	− 2	− 8
10	+46	+ 9	+11	+ 4	− 3	+44	− 7	+ 5	− 6	−10
11	+54·5	+ 3	+13	+11	− 1	+52·5	+ 7	+ 6	+ 2	+ 5
12	+58·5	− 1	+ 7	+ 7	− 2	+56·5	+ 4	+11	+11	+10
13	+56·5	+ 7	+14	+12	+ 8	+54·5	+ 6	+13	+14	+10
14	+49·5	− 5	+ 4	+ 3	0	+47·5	+ 7	+12	+ 7	+ 1
15	+39	− 6	− 8	− 4	−12	+38	+ 1	+11	+10	+ 6
16	+27	−12	− 3	− 5	−11	+26	−15	− 4	− 7	−15
17	+14·5	− 7	−15	−10	−16	+13·5	−39	−41	−46	−50
18	+ 1	−23	−39	−49	−43	+ 0·5	−20	−34	−36	−20
19	−12·5	−17	−10	− 8	− 6	−13	− 7	−15	−13	−11
20	−25·5	+ 2	+ 7	+17	+32	−26·5	− 1	+ 4	+15	+27
21	−38	+ 5	+ 1	+ 7	+18	−39·5	+ 7	+ 9	+17	+19
22	−50	+ 3	+ 5	+13	+17	−51·5	+ 2	+ 9	+ 9	+ 9
23	−60	− 5	+ 4	+10	+ 8	−61·5	+ 5	+ 2	+ 9	+ 7

TABLE IV.

Harmonic Analysis for $-3°$.

log N_0.	sin θ.	cos θ.	sin 2θ.	cos 2θ.	sin 3θ.	cos 3θ.	sin 4θ.	cos 4θ.
0·98	+ ·080	− ·002	− ·010	+0·36	− ·017	− ·006	− ·004	− ·038
1·61	+ ·038	− ·047	− ·034	+ ·117	− ·015	+ ·004	+ ·019	− ·065
2·08	+ ·021	− ·024	− ·059	+ ·118	− ·038	+ ·013	+ ·028	− ·078
2·28	+ ·029	+ ·033	− ·078	+ ·069	− ·084	− ·010	+ ·028	− ·077
1·77	− ·068	+ ·058	− ·093	+ ·034	− ·090	+ ·005	+ ·041	− ·052
1·00	− ·038	+ ·033	− ·053	+ ·019	− ·050	+ ·003	+ ·023	− ·029

Harmonic Analysis for $-5°$.

log N_0.	sin θ.	cos θ.	sin 2θ.	cos 2θ.	sin 3θ.	cos 3θ.	sin 4θ.	cos 4θ.
0·89	+ ·080	+ ·003	+ ·008	+ ·071	− ·069	− ·018	− ·004	− ·041
1·52	+ ·061	− ·034	+ ·010	+ ·103	− ·074	+ ·004	− ·008	− ·092
1·97	+ ·050	+ ·018	+ ·008	+ ·118	− ·088	+ ·010	+ ·019	− ·092
2·18	+ ·048	+ ·013	− ·048	+ ·070	− ·113	+ ·006	+ ·043	− ·053
1·74	− ·043	+ ·062	− ·058	+ ·014	− ·058	+ ·030	+ ·074	− ·012
1·00	− ·025	+ ·036	− ·033	+ ·008	− ·033	+ ·017	+ ·043	− ·007

The first harmonics are thus—

$$\text{for } -3° \quad -·038 \sin\theta + ·033 \cos\theta = -·050 \cos(\theta - 131°)$$
$$\text{for } -5° \quad -·025 \sin\theta + ·036 \cos\theta = -·044 \cos(\theta - 145°)$$

Further comments are reserved until adjacent zones have been analysed.

Counts of the Paris zones $+22°$ and $+24°$ (partially) were made some years ago, but by an oversight never published. Attention was recalled to them by some inquiries of M. Jules Baillaud, who was kind enough to have zone $+23°$ also counted, and to send the results in full detail (for every plate). A selection of these are given below in Tables V. and VI. in hourly sums. The counts of $+24°$ are restricted to two points only which were readily obtained, viz. magnitudes]10·0 (easily recognised by the existence of two figures only instead of three) and the totals. The residuals for zones $+22°$ and $+23°$ are given in Table VII., which corresponds to Table III. above. It seems unnecessary at this stage to give the limited counts for zone $+24°$, as opportunity may be found for completing them later, but the harmonic analysis of them has been subjoined to those of zones $+22°$ and $+23°$ in Table VIII., and it will be seen that the accordance is fair, remembering the limitation of material.

In Tables V. and VI. the second columns, headed]8·0, give the total number of stars not fainter than 8·0. The columns headed 8·5

give the stars from 8·1 to 8·5 : those headed 9·0 give the stars from 8·6 to 9·0, and so on. The last column but one, headed [11·5, gives all the stars recorded as fainter than 11·5. It will be clear from the comparison of this column with the preceding for, say, 13^h and 20^h that the uniformity of the magnitude scale is not above suspicion.

TABLE V.

Counts for Zone +22°. Numbers of Plates, 8, 7, 8, 7, etc.

h	]8·0.	8·5.	9·0.	9·5.	10·0.	10·5.	11·0.	11·5.	[11·5.	All.
0	46	38	47	137	230	326	393	255	141	1613
1	44	28	62	71	191	244	193	79	14	926
2	45	32	43	104	245	377	490	332	285	1953
3	58	36	51	83	146	265	399	381	274	1693
4	59	43	50	97	173	324	443	552	764	2505
5	62	44	73	108	285	463	789	1008	2736	5568
6	93	69	163	174	513	871	1238	1502	1642	6365
7	48	53	80	148	294	489	611	757	889	3369
8	45	39	67	156	249	329	549	557	1914	2905
9	33	32	47	102	186	203	312	349	290	1554
10	47	22	50	87	133	232	240	267	134	1212
11	36	18	56	95	143	162	244	191	63	1008
12	32	19	50	94	157	250	300	356	291	1549
13	36	22	62	80	139	164	179	115	16	813
14	44	31	45	129	188	204	285	130	31	1087
15	40	22	59	149	189	282	376	335	166	1618
16	48	39	66	127	189	244	384	364	337	1798
17	57	43	66	171	304	458	525	640	609	2873
18	78	52	119	175	568	796	1100	1217	823	5028
19	84	69	135	290	449	680	1019	1200	364	4290
20	108	75	133	299	673	1322	1975	2753	4394	11732
21	63	34	80	182	391	619	727	1252	1242	4590
22	46	42	89	147	190	626	692	963	1007	4002
23	48	31	51	98	168	238	335	289	376	1634
Sum	1300	933	1744	3503	6593	10168	13798	15844	17702	71585

TABLE VI.

Counts for Zone +23°. Numbers of Plates, 7, 8, 7, 8, etc.

h	]8·0.	8·5.	9·0.	9·5.	10·0.	10·5.	11·0.	11·5.	[11·5.	All.
0	51	35	94	145	257	485	591	138	0	1796
1	39	37	70	119	222	396	463	403	166	1915
2	34	19	55	115	198	254	271	426	0	1372
3	79	51	72	128	182	261	456	158	4	1391
4	57	23	46	77	130	175	410	308	529	1755
5	57	69	140	241	407	633	1029	1331	1816	5723
6	82	72	124	290	534	744	1340	1574	2781	7541
7	53	73	107	216	450	623	1157	1513	991	5183
8	29	42	61	105	247	298	502	618	375	2277
9	59	40	71	106	241	399	336	350	59	1661
10	43	23	47	90	122	195	192	230	49	991
11	31	40	39	85	111	176	348	98	130	1058
12	39	20	35	53	105	139	200	108	101	800
13	41	45	74	133	170	180	118	35	11	807
14	28	24	64	94	135	316	146	4	0	811
15	46	35	68	164	189	328	82	2	0	914
16	44	42	60	147	215	347	407	343	197	1802
17	55	42	99	211	353	610	709	760	553	3392
18	70	41	110	183	365	620	886	940	1662	4877
19	105	91	140	280	469	720	1102	1441	1879	6227
20	87	58	127	291	440	881	1626	1847	2633	7990
21	77	40	98	263	429	769	1064	1256	1327	5323
22	48	48	95	210	373	587	776	392	131	2660
23	67	42	75	148	344	640	540	328	88	2272
Sum	1321	1052	1971	3894	6688	10776	14751	14603	15482	70538

TABLE VII.

Paris Residuals after Correction for Galactic Latitude.

(The unit is 0·01 in log N_0.)

	Zone +22°.					Zone +23°.				
	Gal. Lat.	8·5.	9·5.	10·5.	All.	Gal. Lat.	8·5.	9·5.	10·5.	All.
h	°					°				
0	−40	+ 5	+ 5	+ 5	− 7	−39	+11	+15	+19	+ 6
1	−38·5	+ 4	− 3	− 2	−27	−37	− 4	− 3	+ 2	0
2	−33·5	− 4	−10	− 1	− 6	−33	−19	− 9	−10	−14
3	−25·5	+ 1	−13	−19	−19	−25	+ 7	− 8	−21	−33
4	−15·5	−14	−30	−35	−24	−15	−22	−38	−49	−32
5	− 4·5	−17	−30	−28	+ 3	− 4	−18	−15	−19	0
6	+ 7·5	− 3	− 1	− 2	+ 5	+ 8	− 1	− 1	− 1	+23
7	+20·5	− 2	− 4	− 2	+ 5	+20·5	0	0	+ 3	+20
8	+33·5	− 1	+ 5	+ 1	+12	+33·5	− 4	− 5	− 2	+11
9	+46·5	+ 3	+ 5	+ 1	+ 3	+46·5	+16	+ 7	+12	+ 3
10	+59·5	+ 5	+ 3	+ 1	− 7	+60	+ 8	+ 6	0	− 6
11	73	0	+13	+ 6	0	+73·5	+ 5	− 1	− 7	− 5
12	+84	− 9	+ 5	+11	+16	+85	+ 3	− 6	− 8	− 8
13	+78	+ 4	+13	+ 6	−13	+78	+15	+22	+ 9	−17
14	+64·5	+10	+14	+ 8	− 9	+65	− 4	+ 8	+13	−14
15	+51·5	+ 2	+19	+14	+ 9	+51·5	+ 7	+16	+10	−23
16	+38·5	+ 6	+ 6	− 4	− 4	+38·5	+11	+11	+ 8	+ 6
17	+25·5	+ 5	+ 5	+ 5	+ 6	+25·5	− 5	+ 3	+ 6	+10
18	+12·5	− 8	− 1	+ 1	+ 2	+12·5	−10	−10	− 9	+10
19	0	− 4	− 2	− 9	−13	+ 0 5	− 1	− 9	−15	0
20	−11·5	+ 6	+ 5	+16	+38	−10·5	− 1	+ 1	+ 3	+28
21	−22	− 1	+ 3	+10	+21	−21	− 3	+ 3	+ 7	+23
22	−31	− 2	+ 4	+17	+23	−30	+ 5	+11	+18	+11
23	−37·5	+ 7	+ 2	− 4	− 3	−36·5	+14	+ 8	+21	+ 9

TABLE VIII.

Harmonic Analysis for Zone +22°.

log N_0.	sin θ.	cos θ.	sin 2θ.	cos 2θ.	sin 3θ.	cos 3θ.	sin 4θ.	cos 4θ.
0·77	− ·035	− ·008	+ ·003	+ ·033	+ ·012	+ ·031	+ ·008	− ·023
1·33	− ·097	− ·067	− ·017	+ ·062	+ ·013	+ ·046	+ ·015	− ·008
1·87	− ·099	− ·031	− ·049	+ ·068	− ·005	+ ·033	+ ·023	− ·005
2·31	− ·065	− ·012	− ·126	− ·016	− ·049	+ ·012	− ·011	+ ·023
2·08	− ·032	+ ·032	− ·161	− ·043	− ·079	− ·032	− ·011	+ ·049
1·00	− ·015	+ ·015	− ·077	− ·021	− ·038	− ·015	− ·005	+ ·024

TABLE VIII.—*continued.*

Harmonic Analysis for Zone +23°.

log N_0.	sin θ.	cos θ.	sin 2θ.	cos 2θ.	sin 3θ.	cos 3θ.	sin 4θ.	cos 4θ.
0·79	−·033	−·039	−·021	+·063	−·007	+·040	−·034	−·006
1·37	−·077	−·037	−·011	+·064	−·028	+·088	−·002	−·011
1·90	−·087	−·009	−·047	+·088	−·022	+·130	+·013	+·006
2·28	−·038	+·041	−·157	−·031	−·019	+·066	+·047	+·052
2·02	−·015	+·108	−·172	−·070	−·006	+·068	+·096	+·075
1·00	−·007	+·053	−·085	−·035	−·003	+·034	+·048	+·037

Harmonic Analysis for Zone +24°.

log N_0.	sin θ.	cos θ.	sin 2θ.	cos 2θ.	sin 3θ.	cos 3θ.	sin 4θ.	cos 4θ.
1·44	−·059	−·071	−·074	+·079	−·028	+·091	+·008	+·012
2·26	−·149	+·013	−·088	−·013	−·036	+·040	+·020	+·036
0·82	−·090	+·084	−·014	−·092	−·008	−·051	+·012	+·024
1·00	−·110	+·103	−·017	−·112	−·010	−·062	+·015	+·029

Attention has chiefly been concentrated on the rate of change per magnitude of the first harmonic, which is deduced in the lowest line of each table and is expressible as follows:—

$$\text{Zone } +22^\circ \quad -\cdot015\sin\theta + \cdot015\cos\theta = -\cdot021\cos(\theta - 135^\circ)$$
$$\text{Zone } +23^\circ \quad -\cdot007\sin\theta + \cdot053\cos\theta = -\cdot053\cos(\theta - 98^\circ)$$
$$\text{Zone } +24^\circ \quad -\cdot110\sin\theta + \cdot103\cos\theta = -\cdot150\cos(\theta - 133^\circ)$$

In the earlier papers of this series, the appropriate correction for Galactic Latitude had not yet been determined, and the uncorrected results gave phases for the first harmonic which fitted the approximate formula—

$$\psi = 247^\circ - 3\cdot66 \times \delta = 161^\circ \text{ for } \delta = 23^\circ\cdot5.$$

The above phases are all considerably less than 161°, as was found for the Bordeaux Zones +14° to +17° when the Galactic correction was applied (*M.N.*, 79, 148). But further comment is reserved until the Oxford Zones (+25° to +31°), which have now been fully reduced, are presented in the corrected form.

Note on the Number of Sensible Stellar Proper Motions to be detected after a given Interval. By F. A. Bellamy, M.A.

1. During the last 15 years, 14 notes on large or sensible proper motions as found from a comparison of plates in the Oxford Astrographic Zones, $+25^{\circ}$ to $+31^{\circ}$, repeated after intervals of 8–24 years, have been presented to the Society. It is possible to form from those results some estimate of the number of proper-motions to be expected after a given interval. Some conclusions of this nature have already been given, especially by the late Mr. R. J. Pocock (*M.N.*, 76, 657). A return to those conclusions will be made presently, but I give first the results I have collected in a rather different way from a larger amount of material, as in Table I.

2. In Table I. are shown in col. 1, the mean intervals in years between the pairs of plates compared. Col. 2, the number of such pairs in the group. Col. 3, the total number of stars examined for p.m. in each group. Col. 4, the number of p.m.'s found for which the combined displacement in x and y remained at least $1''\cdot5$ (R. Int. ·005) after careful remeasurements on both plates. [The rule in our unit is $\Delta x^2+\Delta y^2\geqslant 25$.] In Cols. 5, 6, as some of the groups are small and the accidental errors are large, a smoothing has been adopted by taking the mean of three groups successively; the mean interval in col. 5, 10·0 years, is the weighted mean of 8·4, 9·8, and 10·4, and in col. 6, 3·05 is the sum of 5, 44, and 122 divided by 56; the weights being merely the number of pairs of plates in col. 2. Cols. 7 and 8 give the calculated number of p.m.'s per plate, and the difference from the mean in col. 6.

A plate contains 4·7 square degrees. As 987 square degrees have been examined, they have yielded 1·06 proper-motions per square degree above the limit adopted.

TABLE I.

Collected Results of Proper Motions.

Interval in Years.	No. of Pairs.	Stars examined.	No. of p.m.'s found.	Successive Means of Three Groups.		Calc. No.	O−C.
				Interval.	P.M.'s (per plate).		
8·4	4	1201	5				
9·8	17	3116	44	10·0	3·05	3·07	−0·02
10·4	35	6534	122	10·5	3·52	3·28	+0·24
11·4	17	4187	77	11·1	3·72	3·60	+0·12
12·1	23	3477	80	12·0	4·07	4·01	+0·06
13·5	5	1451	26	13·3	4·17	4·63	−0·46
14·3	25	4750	115	14·8	5·04	5·37	−0·33
15·5	26	4188	141	15·6	5·76	5·77	−0·01
16·6	35	5941	239	16·2	6·20	6·07	+0·13
17·1	6	1554	35	16·8	7·00	6·38	+0·62
20·3	2	333	27	19·3	6·91	7·55	−0·64
22·9	3	534	14	22·7	9·47	9·21	+0·26
23·0	12	1153	120				
Total	210	38419	1045				

3. The means of the groups of three seem to indicate a linear law, and the formula used for the figures in the seventh column is based upon that,

No. of p.m.'s per plate = (0·487 × interval in years) − 1·83 ;

0·487 was obtained from the sum of the differences of p.m. groups multiplied by the differences of intervals and divided by the sum of the squares of the intervals; and the constant − 1·83 is the mean difference of the observed numbers in the p.m. groups and the observed intervals × 0·487.

If this purely empirical law be used for the purpose of extrapolation, then for the 50 years' interval we shall find on each plate 22·5 p.m.'s with a displacement of 1″·5 or greater, or a centennial p.m. of 3″·0. There is reasonable doubt as to whether such extrapolation can be carried on to 100 years with advantage. If it may be extended to that period then we should expect 45 stars per plate of 4·7 square degrees, or ten stars in each square degree, with 1″·5 per century as the minimum limit of proper motion.

4. Though the average number of stars on each plate in the seven zones of the printed Oxford Astrographic Catalogue is 400, it is not applicable in the present instance, because the reason for re-taking some of the plates exposed before 1896, which chiefly form the earlier plate of each pair, was that the plates used between 1892–6 were less sensitive; so that the minimum limit of the magnitudes of the stars examined for this p.m. work was fixed by the older plate and that limit was often not much fainter than 10·5 on the Harvard scale. But with this proviso we may remark that the average number of stars per pair of plates from Table I. is 183; the 45 p.m. stars per plate just mentioned are practically one quarter of this number, so that we may say with some confidence that, according to extrapolation from this linear formula, the majority of the stars given in our seven catalogues will not show any displacement greater than 1″·5 in 100 years.

5. To return now to Mr. Pocock's paper where he used the percentage of p.m.'s for stars examined. Now it seems doubtful whether this is the best way to look at the matter, for it has been shown elsewhere that in the Milky Way, where the total number of stars on a plate is very large, we are not likely to find appreciably more p.m.'s on a single plate than near its Poles, where the total number of stars is very small. The sensible p.m.'s are those of the stars comparatively near us which are apparently equally numerous in all directions.

But by Mr. Pocock's plan one finds the percentage of p.m.'s as $K \times (\text{interval})^2$, where K is a constant; he gave the following results (*M.N.*, **76**, 658):—

Table II.

Interval in years			<10	10·5	11·5	12·5	14·5	15·5	16·5
No. per 1000 stars			14·4	18·8	17·2	24·3	26·2	33·1	44·9
,,	,,	$\times \pi^2$	471	500	382	457	364	407	485
,,	,,	÷(Int.)²(>·144)	·171	·130	·156	·125	·138	·165	

The third line should represent a constant which he took as 440. His τ is the limit of p.m. and varies inversely as the interval. Multiplying by τ^2 in Table II. is equivalent to dividing by the square of the interval, as is given in the fourth line; this also should be constant and has a mean value of about ·150 (= 440 × ·000341).

Using Mr. Pocock's method we may now give a similar table, for comparison of the two methods, using the mean results of groups of three given in Table I.

TABLE III.

Interval in years	10·1	10·5	11·2	12·0	13·3	14·8	15·6	16·2	16·8	19·3	22·7
No. per 1000 stars	15·7	17·6	19·7	20·1	22·8	27·1	33·3	35·5	38·6	31·4	79·8
,, $\times\tau^2$	451	469	463	411	378	365	401	395	401	246	454
,, $\div$(Int.)^2	·154	·160	·158	·140	·129	·124	·137	·135	·137	·084	·155

The 19·3 group has only eleven pairs of plates, and in the 22·7 group most of the plates happen to be near the galactic pole; these have a relatively high number of p.m.'s to stars examined.

The two lines should be constant, but they show a decrease as the interval increases. If we regard the earlier numbers as faulty then the value of Mr. Pocock's constant must be decreased from 440 to about 400 or less; and the other constant as ·136.

To compare with the linear law, we must use the average number of stars per plate, which the above figures give as 183.

TABLE IV.

Comparison of the Two Methods.

Interval in years		10	20	30	40	60	85·7
No. of p.m.'s per 1000 stars	Pocock	13·6	54·4	122·4	217·6	489·6	1000
	Linear	17·5	43·8	70·0	96·2	149·3	217·1

The difference in the two formulæ is strongly marked. At the present time we can re-take regions at Oxford which I first photographed thirty-one years ago; a repetition of such plates now or within the next ten years should decide the point as to which formula is more correct.

The material available is not uniformly distributed in the groups with reference to galactic longitude and latitude, nor is the amount of material similar in the successive groups, and the linear formula used is entirely empirical; but the results given by this linear formula appear to be entitled to some confidence.

Conclusion.

From the collected data of Oxford p.m. work already published some approximate conclusions are drawn as to the probable number of centennial p.m.'s—to the 11 mag. (Harvard scale) and above 1″·5—that may be expected between the declinations + 24° to + 32°.

University Observatory, Oxford:
1923 *May* 9.

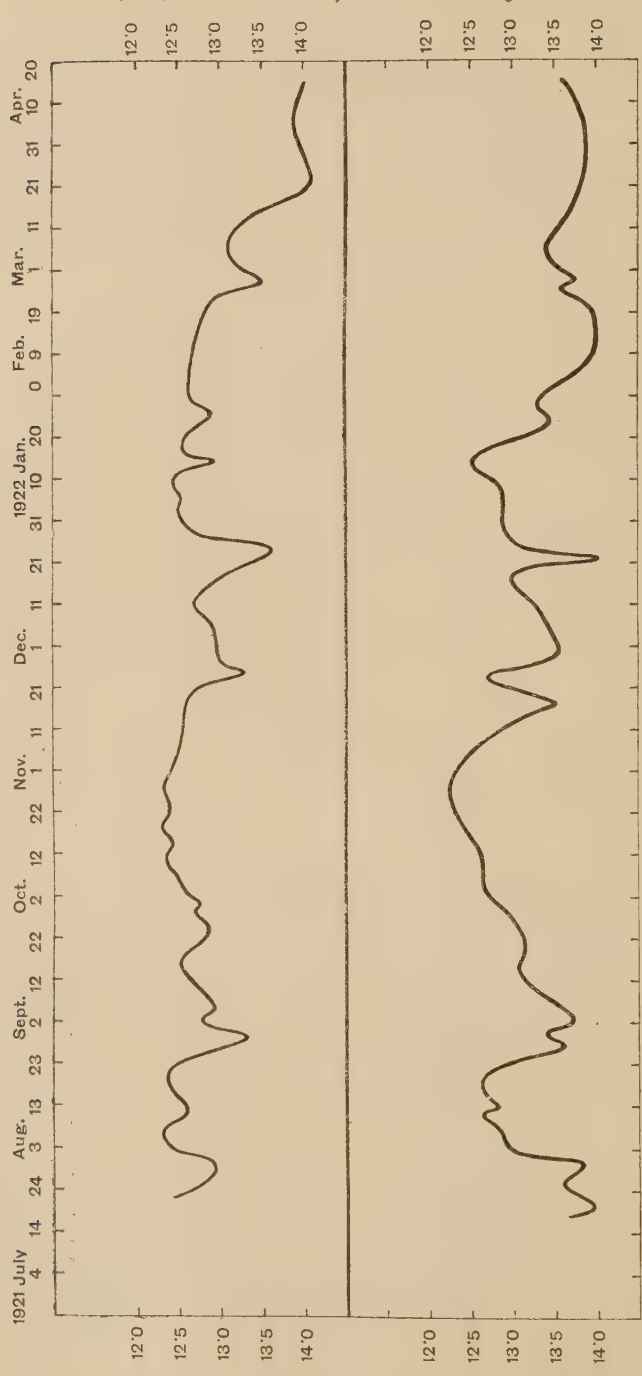
Observations of Nova Persei (1901) *in* 1922–23.

By W. H. Steavenson.

The results of photometric observations of this nova for the apparition of 1921–22 were given in *M.N.*, **82**, 403. The present paper covers the period 1922 July 15 to 1923 April 14, during which the nova was observed on 102 nights. Nearly all the observations were made with the 6-inch refractor, and most of the estimates are probably within about $0^m\cdot1$, though a few, indicated as approximate only, may be as much as $0^m\cdot25$ in error.

Following are the individual observations :—

Date.	Mag.	Date.	Mag.	Date.	Mag.
1922 Jul. 15	<13·2	1922 Oct. 13	12·69	1923 Jan. 3	12·96
18	13·6±	14	12·64	6	12·91
19	14·07	15	12·64	8	12·91
20	13·88	16	12·64	9	12·96
21	13·96	17	12·43	10	12·79
25	13·6±	23	12·32	11	12·79±
30	13·86	24	12·30	14	12·53
31	13·86	25	12·37	15	12·64±
Aug. 1	13·06	26	12·32±	16	12·64
7	12 77	27	12·22	20	13·22
8	12·97	28	12·32	22	13·35
15	12·74	30	12·22	23	13·42
16	12·79	Nov. 1	12·48	24	13·52
17	12·74	4	12·43	25	13·42
20	12·79	6	12·62	27	13·33±
23	12·98	7	12·54	30	<13·2
24	13·05	9	12·69±	Feb. 5	13·82
26	13·59	17	13·57	6	13·88
27	13·63	24	12·72	8	14·0±
31	13·57	25	13·25	11	13·96
Sept. 7	<13·2	26	13·25	16	13·99
9	<13·2	27	13·13	18	14·03
13	13·26	28	13·58	21	14·00
15	13·20	Dec. 5	13·52	25	13·52
17	13·20	6	13·4±	26	13·83
23	13·44	17	12·99	27	<13·2
26	13·10	19	13·13	Mar. 6	13·43
29	13·05	20	13·18	10	13·57
Oct. 1	12·90	22	14·07	17	13·77
3	12·74	23	13·28	18	13·67±
6	12·74	24	13·33	24	<13·5
9	12·74	26	13·03	26	13·9±
11	12·74	27	12·94	Apr. 4	13·7±
12	12·69	1923 Jan. 2	12·79	14	13·7±


Light-Curves of Nova Persei (1901).
Upper = 1921-22.
Lower = 1922-23.
1921 July 4 14 24 Aug. 3 13 23 Sept. 2 12 22 Oct. 2 12 22 Nov. 1 11 21 Dec. 1 11 21 31 1922 Jan. 10 20 Feb. 9 19 Mar. 1 11 21 31 Apr. 10 20
1922 July 4 14 24 Aug. 3 13 23 Sept. 2 12 22 Oct. 2 12 22 Nov. 1 11 21 Dec. 1 11 21 31 1923 Jan. 10 20 30 Feb. 9 19 Mar. 1 11 21 31 Apr. 10 0
12·0
12·5
13·0
13·5
14·0

As will be seen from the above figures, the nova continues to exhibit marked variability of light. The general form of the light curve for 1922–23 is shown in the lower half of the accompanying figure. Above it, for the sake of comparison, is given the curve for 1921–22, based on a combination of the writer's observations with those of the late Professor Barnard (*M.N.*, 82, 457).

There is a certain general similarity between the two curves which, though far from rigid, seems to be too close to be altogether accidental. Smoothed of minor fluctuations of short duration, both curves show—

(*a*) A maximum about the middle of August,
(*b*) A rapid fall to minimum at end of August,
(*c*) A slow rise to maximum late in October,
(*d*) A minimum in late November,
(*e*) A maximum early in January,
(*f*) A slow fall to minimum near the end of March, with a tendency towards a general dying-away of the oscillations.

Were it not for the minor fluctuations, the curves would be still more alike. As Barnard has remarked, there is no doubt of the reality of these short-lived oscillations, but it is perhaps still too early to identify them with certainty as being truly periodic. It looks very much as if they were nearly all in the nature of *downward* interruptions of what would otherwise be a smooth long-period variation. The presence of occulting matter in revolution round the nova suggests itself as a possible explanation, but a much closer study of the light-curve is needed before any definite theory can be built up. Meanwhile, the spacing of the following temporary minima may be of some significance :—

1921–22.	1922–23.	(Interval $\pm 1^D$).
August 29	August 27	363^D
September 23	September 22	364
December 24	December 22	363
January 12	January 9	362
January 26	January 24	363

These short fluctuations would seem worthy of closer study. What is required is a record that shall be, if possible, complete for *every day* of the nine months during which good observations are possible. Observers of variable stars seem reluctant to take up the study of this extremely interesting object, and the much-lamented death of Professor Barnard has greatly blackened the outlook. Perhaps it is not too much to hope that at least two or three new observers will step into the breach for the coming apparition of the nova. A complete list of comparison stars, with a chart of the field, will be found in *M.N.*, 86, 711.

Mean Areas and Heliographic Latitudes of Sunspots in the year 1921.

(*Communicated by the Astronomer Royal.*)

The results here given are in continuation of those in the *Monthly Notices*, 83, 345, and are derived from the measurement of photographs taken at the Royal Observatories of Greenwich and the Cape, and in India at Kodaikánal Observatory and at Dehra Dûn.

Table I. gives the mean daily area of umbræ, whole spots, and faculæ for each synodic rotation of the Sun for the year 1921, and Table II. gives the same particulars for each entire year from 1913 to 1921 inclusive. The areas are given in two forms: firstly, projected areas, *i.e.* as seen and measured on the photographs, these being expressed in millionths of the Sun's apparent disc; secondly, areas corrected for foreshortening, the areas in this case being expressed in millionths of the Sun's visible hemisphere.

Table III. gives for each rotation in the year 1921 the mean daily area of the whole spots (corrected for foreshortening) and the mean heliographic latitude of the spotted areas for spots north and for spots south of the equator, together with the mean heliographic latitude of the entire spotted area and the mean distance from the equator of all spots. Table IV. gives the same information for each year as a whole from 1913 to 1921. Tables II. and IV. are thus in continuation of similar tables for the years 1874 to 1888 on pp. 381 and 382 of vol. xlix. of the *Monthly Notices*; for the years 1889 to 1902 on pp. 465 and 466 of vol. lxiii.; and for the years 1901 to 1914 on pp. 402 and 403 of vol. lxxvi.

The rotations in Tables I. and III. are numbered in continuation of Carrington's series (*Observations of Solar Spots made at Redhill*, by R. C. Carrington, F.R.S.), No. 1 being the rotation commencing 1853 November 9. The assumed prime meridian is that which passed through the ascending node at mean noon of 1854 January 1, and the assumed period of the Sun's sidereal rotation is 25·38 days.

The principal features of the record for the year 1921 are as follows:—

1. The continued decrease in the mean daily spotted area to 420 units (corrected for foreshortening) from 618 units for the previous year and 1537 units for the maximum year of this cycle.

2. The amount of faculæ shows a decrease concurrently with the spots. There is a steady falling off in activity during the latter half of the year.

3. Comparing the two hemispheres, the greater spot activity is shown by the northern. During the year 1920, the southern hemisphere showed for the first time this cycle a preponderance in spot activity, which has thus in 1921 reverted to the northern hemisphere. It may be remarked that the northern hemisphere was markedly quiescent during Rotation 901 and the southern during Rotations 905 and 910.

4. The mean latitude of the spots shows a further progression equatorwards. The large naked-eye group on the Sun's central meridian on May 14 was exactly on the solar equator.

5. During the latter half of the year the bulk of the spot groups were confined to one-half of the Sun, between longitudes 320° – 0° – 130°. From longitudes 120° to 325° in the northern hemisphere only six very small groups were seen during this period.

6. On 31 days no spots were visible on the disc, as compared with 4 days in the previous year.

7. The number of separate groups lasting for at least two days was 129, as compared with 187 for the year 1920. Of the 129, 70 groups were in the northern and 59 in the southern hemisphere, including 4 which were within 1° of the equator. Besides these, there were recorded 80 small groups (mostly consisting of one or two very small spots) visible for one day only; 42 were in the northern and 38 in the southern hemisphere.

TABLE I.

No. of Rotation.	Rotation commenced G.M.T. (Civil).		No. of Days on which Sun photographed.	Mean of Daily Areas. Projected. Umbræ.	Whole Spots.	Faculæ.	Corrected for Foreshortening. Umbræ.	Whole Spots.	Faculæ.
	1920-21	d							
900	Dec.	29·41	27	162	785	949	113	586	1094
901	Jan.	25·75	27	77	383	755	51	254	859
902	Feb.	22·09	27	53	266	840	44	237	939
903	Mar.	21·41	28	136	727	775	104	576	876
904	Apr.	17·69	27	147	790	751	105	600	904
905	May	14·93	27	110	528	783	80	402	899
906	June	11·14	27	188	1014	866	129	702	987
907	July	8·34	28	99	492	734	67	331	796
908	Aug.	4·55	27	73	391	463	51	267	539
909	Aug.	31·79	27	95	423	512	68	328	621
910	Sept.	28·06	27	42	200	426	33	167	485
911	Oct.	25·34	28	125	664	301	92	504	360
912	Nov.	21·65	27	138	786	366	96	547	414

TABLE II.

Year.	No. of Days on which Sun photographed.	Projected. Umbræ.	Whole Spots.	Faculæ.	Corrected for Foreshortening. Umbræ.	Whole Spots.	Faculæ.
1913	365	2	10	87	1	7	95
1914	365	40	195	396	31	152	454
1915	365	186	920	1289	137	697	1521
1916	365	186	978	1512	135	724	1785
1917	365	346	2101	1882	247	1537	2305
1918	364	255	1504	1526	188	1118	1882
1919	365	253	1437	1445	182	1052	1729
1920	366	156	837	1052	112	618	1219
1921	365	110	571	647	79	420	743

TABLE III.

No. of Rotation.	Rotation commenced G.M.T. (Civil).	Spots NORTH of the Equator. Mean of Daily Areas.	Spots NORTH of the Equator. Mean Heliographic Latitude.	Spots SOUTH of the Equator. Mean of Daily Areas.	Spots SOUTH of the Equator. Mean Heliographic Latitude.	Mean Heliographic Latitude of entire Spotted Area.	Mean Distance from Equator of all Spots.
	1920–21 d		°		°	°	°
900	Dec. 29·41	118	5·17	468	10·93	− 7·69	9·77
901	Jan. 25·75	7	10·64	247	11·57	−11·00	11·55
902	Feb. 22·09	83	8·30	154	8·92	− 2·92	8·70
903	Mar. 21·41	455	10·89	121	9·41	+ 6·62	10·58
904	Apr. 17·69	527	3·16	72	14·64	+ 1·02	4·54
905	May 14·93	381	2·98	21	7·77	+ 2·42	3·23
906	June 11·14	490	9·79	212	7·45	+ 4·58	9·09
907	July 8·34	116	8·24	216	9·20	− 3·11	8·87
908	Aug. 4·55	155	11·05	112	7·08	+ 3·46	9·39
909	Aug. 31·79	255	9·80	73	8·16	+ 5·80	9·44
910	Sept. 28·06	164	6·58	3	10·37	+ 6·28	6·65
911	Oct. 25·34	435	6·40	69	6·12	+ 4·68	6·36
912	Nov. 21·65	272	7·62	275	5·76	+ 0·90	6·69

TABLE IV.

Year.	No. of Days on which Sun photographed.	Spots NORTH of the Equator. Mean of Daily Areas.	Spots NORTH of the Equator. Mean Heliographic Latitude.	Spots SOUTH of the Equator. Mean of Daily Areas.	Spots SOUTH of the Equator. Mean Heliographic Latitude.	Mean Heliographic Latitude of entire Spotted Area.	Mean Distance from Equator of all Spots.
			°		°	°	°
1913	365	5	24·81	2½	20·08	+9·81	23·23
1914	365	99	21·11	53	23·06	+5·71	21·79
1915	365	379	17·95	318	19·75	+0·77	18·77
1916	365	470	15·14	254	17·05	+3·83	15·81
1917	365	860	13·82	677	15·66	+0·84	14·63
1918	364	609	11·97	509	13·69	+0·29	12·75
1919	365	559	9·85	493	11·79	−0·28	10·76
1920	366	208	11·31	410	9·98	−2·83	10·43
1921	365	261	7·27	159	8·92	+1·13	7·90

Royal Observatory, Greenwich:
1923 *May* 4.

The Intensities of Absorption Lines in Stellar Spectra, and the Temperatures and Pressures in the Reversing Layers of Stars. By R. H. Fowler and E. A. Milne. (Plates 15, 16.)

(*Communicated by the Director of the Solar Physics Observatory, Cambridge.*)

§ 1. *Descriptive Introduction and Summary.*—Saha's theory of high-temperature ionisation has been successful in accounting for many of the principal features of stellar spectra. It has dealt chiefly with the relative intensities of arc lines and enhanced lines under various conditions of temperature and pressure. Its fundamental principle is that, since arc lines can be produced only by the neutral atom, enhanced lines only by the ionised atom, the relative intensities of the two kinds of lines must give some indication of the relative numbers of neutral and ionised atoms. Further, since under given conditions of temperature and pressure these relative numbers can be calculated from the theory of dissociative equilibrium, evidence is obtained as to the actual temperatures and pressures in the reversing layers of stars.

The applications of the existing theory may be divided into two main groups. The first is typified by comparisons between the spectra of the solar chromosphere and the reversing layer, between the spectra of the reversing layer and sun-spots, and between the spectra of giant and dwarf stars of the same spectral type. It is shown that the intensity differences are largely explained by changes in the degree of ionisation resulting either from temperature differences (sun and spot) or from pressure differences (chromosphere and reversing layer, giant and dwarf). Such applications have been made in detail by Saha * and Russell.† In the second group attention is devoted to the general march of the intensity of a given line or group of lines through the sequence of stellar spectral types, and an attempt is made to deduce the temperature scale. The scale is fixed by noting when a particular absorption line is on the point of appearing or disappearing. We shall refer to either of these as the "marginal appearance" of the line. At such a point in the sequence the fraction of atoms capable of absorbing the line must be very small, and, assuming a value for the pressure, the temperature can be calculated. Such calculations have been made by Saha,‡ and recently by H. H. Plaskett.§

But the precision of calculations of this kind has been hitherto questionable, owing to the difficulty of formulating the conditions for the marginal appearance of a line.‖ Firstly, we do not know how small the "very small" fraction of atoms must be to give the marginal appearance. Secondly, the point of marginal appearance will depend on the relative abundance of the element giving the line—lines due to a more abundant element will persist to a smaller relative concentration.

* *Phil. Mag.*, **40**, 472 (1920); **40**, 809 (1920).

† *Astrophys. Journ.*, **55**, 119 (1922).

‡ *Proc. Roy. Soc.*, **A**, **99**, 135 (1921); *Zeits. für Phys.*, **6**, 40 (1921).

§ "The Spectra of Three O-Type Stars," *Pub. Domin. Astrophys. Obs.*, **1**, No. 30, (1922).

‖ Some of the difficulties have been discussed in a recent article by one of us Milne, *Observatory*, **46**, 113 (1923).

There is a similar difficulty connected with the relative efficiency of various atoms as absorbers. Large uncertainties are liable to arise from all these sources.

However, the general qualitative adequacy of the theory inclines one to believe that imperfections of this kind are imperfections of our knowledge and not of the theory; it is reasonable to conclude that in the main the intensities of absorption * lines depend on the numbers of atoms capable of yielding them. Let us then make the definite assumption that other things being equal *the intensity of a given absorption line in a stellar spectrum is proportional to the concentration of atoms in the stellar atmosphere capable of absorbing the line.* It is then one of the main objects of this paper to point out that the foregoing difficulties are avoided if we devote attention to the place in the stellar sequence at which a given line attains not its marginal appearance but its *maximum intensity.* The maximum intensity will be attained at the maximum concentration of atoms capable of absorbing the line, and the conditions therefore involve only the temperature and pressure; the relative abundance of the elements obviously does not enter, nor is a knowledge required of the total number of atoms producing the line or of their efficiency as absorbers. The temperature at which, for a given pressure, a given line whose series relations are known attains its maximum is provided by pure statistical theory. Consequently, each observed maximum of a line in the stellar sequence gives a certain relation between temperature and pressure holding at the point in the sequence concerned. We submit that it is only in this way that Saha's theory can be fairly applied and strictly tested, in the first instance, as regards the quantitative determination of stellar pressures and the scale of stellar temperatures, and we undertake such a preliminary application here. When the proper foundations of the theory have been laid, a discussion of marginal appearances should later provide important information as to relative abundance and the minimum number of atoms necessary for appearance.

It is easy to calculate from Saha's theory as it stands the condition for the maximum intensity of lines like the H and K lines of calcium. These lines are the first pair of the principal series of Ca^+. As the temperature of Ca vapour increases the concentration of Ca^+ atoms steadily increases until (at a point where the proportion of neutral atoms is very small) second-stage ionisation sets in and the concentration of Ca^+ atoms diminishes.

But Saha's theory has not hitherto accounted quantitatively for the maxima of such lines as the Balmer lines of hydrogen, the arc lines of helium, or the line $\lambda 4481$ Mg^+. Before an H atom can absorb a Balmer line the electron must be lifted into a 2-quantum orbit; similarly the so-called arc lines of He are not absorbed by the normal atom, but only by an atom in which one electron has been shifted to an orbit of higher quantum number. Saha pointed out † that as the temperature increases

* It should be emphasised that the physical conditions of emission spectra are still so far from clear that we cannot hope to include in the theory at this stage any bright-line stellar spectra.

† *Proc. Roy. Soc.*, loc. cit., p. 141.

the fraction of atoms in the higher quantum states will increase, but stated that he could give no definite calculation.* We speak of an atom not in its normal state as an "excited" atom. It is the second main object of this paper to point out that with the aid of the general theory of assemblies of atoms, electrons and radiation in statistical equilibrium,† it is possible to determine the fraction of excited atoms under given conditions of temperature and pressure and to use this to discuss the intensities of lines such as those of the Balmer series.

Lines due to the neutral atom in its normal state—that is to say, lines of the principal series ‡ of the neutral atom—should always decrease in intensity as we traverse the spectral types from M to higher temperatures: for the fraction in the normal state (initially practically unity) can only decrease as the temperature increases.§ It is easy to see that all other series of lines should have a maximum intensity somewhere. Consider first a subordinate series of a neutral atom. The fraction of atoms capable of absorbing this series—that is, the fraction of atoms in, say, the n-quantum state—is the product of two factors: (1) the fraction of the atoms still neutral (the un-ionised fraction), and (2) the fraction of these neutral atoms which are in the n-quantum state. The first factor decreases steadily from 1 to 0 as the temperature rises, whilst the second increases similarly from a very small value, practically zero. The product must consequently somewhere have a maximum. In this paper we identify the theoretical maximum concentration of excited atoms in a given state with the observed stellar maximum in the intensity of the corresponding absorption series. ‖

The subordinate series of an ionised atom behave ultimately in the

* On p. 151 of the paper cited he gives 12,000° (A0 stars) as the temperature of maximum concentration of the 2-quantum orbits of H, and 17,000° (B2 stars) as that of the 1π-orbits of He. It appears, however, that these are not calculated temperatures. From marginal appearances he has already found 12,000° for A0 stars (marginal appearance of He arc lines) and 17,000° for B2 stars (marginal appearance of λ4686 He^+). He gives the maximum proportion of Ca^+ atoms as occurring at 7000°. It is not clear, however, how he obtains a definite temperature for the marginal appearance of the Balmer lines (2-quantum orbits just appearing, 4500°, Mb stars) or of the He arc lines (1π-orbits just appearing, 12,000°, A0 stars), without a calculation of the numbers of atoms in these higher quantum states. [Saha's $2p$ is here called 1π in conformity with the notation of this paper.]

† Darwin and Fowler, *Phil. Mag.*, **44**, 450 (1922); **44**, 823 (1922). Fowler, *Phil. Mag.*, **45**, 1 (1923).

‡ In this connection the term "principal series" must strictly be interpreted always as "the normal absorption series of the cold vapour," which is not always (*e.g.* in the Al sub-group) the principal series ($1s - mp$) of spectroscopy. The normal Al atom absorbs the sharp and diffuse series, not the principal series.

§ Though true in general for relevant temperatures, this will be modified at low temperatures, for then other circumstances such as condensation or combination of the vapour will arise, and the theory no longer applies in this simple form. But such considerations are not usually of importance, and need not be again referred to here.

‖ In making this identification we tacitly introduce the further very natural assumption that the material constitution of the stars as regards the particular element considered—that is, the absolute abundance of the element—changes but slowly along the stellar sequence, if at all, in the neighbourhood of the maximum. It is clear that some assumption of this kind is necessary. For otherwise, for example, the maximum in the intensity of the Balmer lines might conceivably be due to a maximum in the absolute abundance of hydrogen.

same way as those of a neutral atom. We have already seen that the principal series of an ionised atom should have a maximum. This covers all cases. It will be noticed that the maximum of the principal series of an ionised atom arises in a way different from the maximum of any subordinate series; it is, in fact, much blunter. There is thus a sharp distinction between principal and subordinate series as regards their behaviour in stellar spectra.* For the purpose of investigations such as the present subordinate series are of much the greater importance.†

The theory provides a definite relation between the pressure and temperature at a maximum, provided the series relations are fully known. We find it convenient in the first instance to calculate the pressures for maxima at given temperatures. The general result of the comparison with observation is that the observed and theoretical maxima can be fitted together into a single consistent scheme, provided the pressures in reversing layers are of the order of 10^{-4} atmos. It then appears that, if this is also the pressure in the reversing layer contributing the He lines in early B-type stars, then stars of type B2 must have a temperature of 16,000°–17,000°, a temperature higher than is often supposed ‡—in particular higher than the temperatures deduced from spectro-photometric observations or from colour-indices. It is of course precisely in this region of high temperature, where the maximum intensity of the continuous spectrum lies in the ultra-violet, that the shape of the intensity curve in the visual-photographic region becomes insensitive as an indicator of temperature. The present investigation seems to demand a more extended temperature scale for all early-type stars.

A word must be said as to the hypothesis of statistical (or thermodynamic) equilibrium, which is the foundation of the theory. In the first place the equilibrium of the reversing layer is in no way altered by the presence of temperature radiation of its own temperature. § It can only be disturbed by the radial temperature gradient which results in a flux of radiation of a different temperature. The effect may be important in certain cases, ‖ but it takes its natural place as a correction to the first approximation (namely, approximate thermal equilibrium), and need not be appealed to till a full discussion has shown up the insufficiencies in the simple theory. Further, it must be borne in mind that all statistical equilibria are by their very nature independent of

* H. N. Russell, *loc. cit.*, p. 138, etc., has pointed out the difference to be expected between lines of principal and subordinate series in comparing sun and spot.

† A specific case of the formation of outer levels due solely to rise of temperature and the consequent appearance of a further series of absorption lines is afforded by recent observations of Grotrian on the vapour of thallium (*Zeit. für Phys.*, 12, 218, (1922)). The cold vapour absorbs the short wave-length components of the lines of the sharp and diffuse series (Tl being a member of the Al sub-group). The line $1\pi_2 - 1\sigma$ appears reversed at 400°, the line $1\pi_1 - 1\sigma$ at 800°, at which temperature $1\pi_1$ levels have made their appearance.

‡ Saha (*loc. cit.*) has also given temperatures of this order for these stars (*v. supra*). His estimate, however, is based on pressures of the order of 1 or 10^{-1} atmos., and is liable to all the uncertainties arising from determinations by marginal appearances, so that the agreement is somewhat fortuitous.

§ Darwin and Fowler, *Proc. Camb. Phil. Soc.*, 21, 262 (1922).

‖ Russell, *Astrophys. Journ.*, 55, 354 (1922). K. T. Compton, *Phys. Rev.*, 20, 283 (1922).

all mechanisms of energy exchange. Only when deviations from such equilibria have been established does the consideration of particular mechanisms become relevant. Finally, it is in no way contended that the hypothesis of thermal equilibrium is adequate to account for all the facts.

A somewhat fuller treatment of statistical equilibrium is required than is given in previous presentations. There are certain inadequacies in the usual (Saha's) thermodynamic treatment which require more detailed analysis. This has been done in a recent paper by one of us,* in a form suitable for application here.

The arrangement of the present paper may thus be summarised. In § 2 we obtain formulæ for the theoretical maxima of the different types of series, and in § 3 we apply the results to the observations in actual cases. The atoms considered are H, He, HeI+, Na, Mg, MgI+, Ca, CaI+, SrI+, Zn, Ba,I+ and *every series of lines is included for which we have been able to find suitable data.* In view of the low pressures to which we are led, we consider shortly in § 4 other lines of evidence which have recently converged to the view that normal reversing layer pressures are very small. We collate these results as they bear on the stellar temperature scale in § 5. In § 6 we point out shortly various facts which appear to be accounted for or not to be accounted for by the theory in its present stage.

§ 2. *The Statistical Theory of the Maxima of Absorption Lines.*—We require first two formulæ: (*a*) a formula for the fraction of atoms in a given state of ionisation, (*b*) a formula for the fraction of these in a given quantum-state.

Let the numerals 1, 2, . . . denote the possible stationary states of a given neutral atom, arranged in order of decreasing negative energy; the numeral 1 denotes the atom in its normal (unexcited) state. Let $\chi_1, \chi_2 \ldots$ denote the energies of the levels, so that χ_1 is the ionisation potential, $\chi_1 - \chi_2$ the first resonance potential, etc. Further, let $q_1, q_2 \ldots$ be the "weights" † of the corresponding states. It is necessary to introduce the "partition function" $b(\mathrm{T})$, defined by

$$b(\mathrm{T}) = q_1 + q_2 e^{-(\chi_1 - \chi_2)/k\mathrm{T}} + q_3 e^{-(\chi_1 - \chi_3)/k\mathrm{T}} + \ldots,$$

where k is Boltzmann's constant and T is the temperature.

Let x be the fraction of atoms ionised; let P_e be the partial pressure of electrons from all sources; and let σ be the number of valency electrons in the atom in equivalent orbits. Then the ionisation is given by the formula ‡

$$\log \frac{x}{1-x}\mathrm{P}_e = -\frac{\chi_1}{k\mathrm{T}} + \tfrac{5}{2}\log \mathrm{T} + \log \frac{(2\pi m)^{\frac{3}{2}}k^{\frac{5}{2}}\sigma}{h^3} - \log b(\mathrm{T}) \qquad . \quad (1)$$

where m is the mass of the electron and h is Planck's constant.

* Fowler, *Phil. Mag.*, **45**, 1 (1923).

† The usual notation is $p_1, p_2 \ldots$, but we have changed it in order to avoid confusion with the "p" terms ($1p_2$, etc.) of spectral series.

‡ Fowler, *loc. cit.*, equation (7·4). It has been modified in two ways, first by the introduction of P_e instead of the total pressure, following Russell, *Astrophys. Journ.*, **55**, 120 (1922); secondly, by the introduction of the factor σ. This occurs owing to the fact that any one of the σ equivalent electrons may be removed in ionisation. In our applications $\sigma = 1$ or 2.

This differs from Saha's formula by the terms in σ and $b(\mathrm{T})$. It is convenient here to give the values of the q's in the various cases that arise. For hydrogen and ionised helium, according to Bohr, *

$$q_1 = 1.2, \quad q_2 = 2.3, \quad q_r = r(r+1).$$

(For these atoms $\chi_r = \mathrm{N}/r^2$ and $\chi_r = 4\mathrm{N}/r^2$ respectively, where N is Rydberg's constant.) For all monovalent atoms other than hydrogen,

$$q_1 = q_2 = \ldots = q_r = 1.$$

For divalent atoms (alkaline earths) and helium,

$$q_1 = 1, \quad q_2 = q_3 = \ldots = q_r = 2.$$

(In this case either of the two valency electrons can be excited.) With these values of the q's the series for $b(\mathrm{T})$ *diverges*, † in all cases, owing to the theoretically infinite number of stationary states. This is a real difficulty in the existing theory, but for physical reasons one must suppose the series effectively cut off after a certain number of terms. Usually the series then reduces (as regards its numerical value) practically to its first term. Thus Saha's formula, as regards the omission of $b(\mathrm{T})$, is only in error when $q_1 \neq 1$, *i.e.* for H and He^+.

Let f_r be the fraction of neutral atoms which are in the r-state. This is given by the formula ‡

$$f_r = q_r e^{-(\chi_1 - \chi_r)/k\mathrm{T}} / b(\mathrm{T}) \quad . \quad . \quad . \quad . \quad (2)$$

The fraction n_r of the total number of atoms which are neutral and in the r-state is then

$$n_r = f_r(1 - x).$$

From equation (1) we find

$$1 - x = \frac{b(\mathrm{T})}{b(\mathrm{T}) + a\mathrm{T}^{\frac{5}{2}} e^{-\chi_1/k\mathrm{T}}} \quad . \quad . \quad . \quad . \quad (3)$$

where

$$a = \frac{(2\pi m)^{\frac{3}{2}} k^5 \sigma}{h^3 \mathrm{P}_e} = \frac{0{\cdot}332\sigma}{\mathrm{P}_e},$$

on inserting the values $h = 6{\cdot}55 \times 10^{-27}$, $k = 1{\cdot}37 \times 10^{-16}$, $m = 9{\cdot}00 \times 10^{-28}$. Hence

$$n_r = \frac{q_r e^{-(\chi_1 - \chi_r)/k\mathrm{T}}}{b(\mathrm{T}) + a\mathrm{T}^{\frac{5}{2}} e^{-\chi_1/k\mathrm{T}}} \quad . \quad . \quad . \quad . \quad (4)$$

The fraction n_r will be a maximum, and with it by hypothesis the intensity of the series of absorption lines for which the initial stationary state is the state r, at the temperature for which

$$\frac{e^{\chi_r/k\mathrm{T}}}{b(\mathrm{T})e^{\chi_1/k\mathrm{T}} + a\mathrm{T}^{\frac{5}{2}}}$$

* Bohr, "The Quantum Theory of Line Spectra," *Mem. Danish Acad.*, **4**, No. 1, part 2, p. 76.

† Fowler, *loc. cit.*, p. 19.

‡ *Loc. cit.*, p. 18, equation (6·31).

is a maximum. Differentiating with regard to T (ignoring the variation of $b(T)$), equating to zero and solving the resulting equation for P_e, we find that the maximum occurs * when

$$P_e = \frac{0{\cdot}332\sigma}{b(T)} \frac{\chi_r + \frac{5}{2}kT}{\chi_1 - \chi_r} T^{\frac{5}{2}} e^{-\chi_1/kT} \quad . \quad . \quad . \quad (5)$$

From (5) and (3) the degree of ionisation at the maximum is readily found to be given by either of the simple formulæ

$$x = \frac{\chi_1 - \chi_r}{\chi_1 + \frac{5}{2}kT_{max}}, \qquad 1 - x = \frac{\chi_r + \frac{5}{2}kT_{max}}{\chi_1 + \frac{5}{2}kT_{max}} \quad . \quad . \quad (6)$$

The essential difference between principal series ($\chi_r = \chi_1$) and subordinate series ($\chi_r \neq \chi_1$), which has been already pointed out, is clearly shown by formulæ (6); for the former the maximum must occur at $x = 0$, that is, theoretically at zero temperature.

Equation (5) gives the value of the partial electron pressure P_e for the maximum of a given series to occur at a given temperature; P_e is larger the larger is T_{max} and the smaller is $\chi_1 - \chi_r$. In thus regarding P_e as fundamental we are in effect assuming that, due to the presence of more easily ionised atoms, there are so many electrons present that the partial electron pressure is practically independent of the degree of ionisation of the element under discussion. In all important cases P_e will only differ from the total pressure by a factor whose extreme values are perhaps 1 and $\frac{1}{3}$.

So far the results have been enunciated in a form which applies directly only to first-stage ionisation. It has been assumed that at the relevant temperatures second-stage ionisation has not yet begun; owing to the difference in value between first-stage and second-stage ionisation potentials this is very closely true. (We have also assumed that the degrees of freedom of the atom other than those of the last electron are completely unexcited—that is, that there is never more than one electron in process of coming loose at any one time.) To extend the results to further ionisation, we simply observe that, after the number of neutral atoms remaining can be neglected, the above work applies as it stands to the exciting of the ionised atom and the development of second-stage ionisation. A short calculation shows that the neutral atoms can be neglected in all such calculations, and the formulæ already obtained apply equally to the behaviour of subordinate series of ionised or neutral atoms.

To discuss the behaviour of the principal series of an ionised atom we must examine further the intermediate region. If dashes refer throughout to second-stage ionisation and excitation, then in place of

* It will be observed that in determining the maximum we keep P_e constant. In a future refinement of the theory it might be necessary to take account of the fact that the pressure in a reversing layer is probably a function of spectral type and different for giants and dwarfs. This could easily be done by finding the maximum of (4) subject to a relation $f(T, P_e) = 0$.

(1) we have the two equations

$$\log \frac{x}{1-x-x'} P_e = -\frac{\chi_1}{kT} + \tfrac{5}{2}\log T + \log \frac{(2\pi m)^{\frac{3}{2}}k^{\frac{5}{2}}\sigma}{h^3} - \log b(T) \quad (7)$$

$$\log \frac{x'}{x} P_e = -\frac{\chi_1'}{kT} + \tfrac{5}{2}\log T + \log \frac{(2\pi m)^{\frac{3}{2}}k^{\frac{5}{2}}\sigma'}{h^3} - \log b'(T) \quad (8)$$

On solving for x we obtain

$$\frac{1}{x} = 1 + \frac{b(T)}{a} T^{-\frac{5}{2}} e^{\chi_1/kT} + \frac{a'}{b'(T)} T^{\frac{5}{2}} e^{-\chi_1'/kT} \quad . \quad . \quad (9)$$

Since x is the fraction of ionised atoms, the fraction with their last electron in its normal state is $q_1'x/b'(T)$, and the maximum development of the principal series will occur when $dx/dT = 0$ (neglecting the variation of $b'(T)$). This leads to

$$P_e = 0{\cdot}332 \left[\frac{\sigma\sigma'}{b(T)b'(T)}\right]^{\frac{1}{2}} \left[\frac{\chi_1' + \frac{5}{2}kT}{\chi_1 + \frac{5}{2}kT}\right]^{\frac{1}{2}} T^{\frac{3}{2}} e^{-\frac{1}{2}(\chi_1+\chi_1')/kT} \quad . \quad (10)$$

Calculations of concentrations of ionised atoms according to (9) give the very flat maximum already referred to.

All our formulæ give P_e in dynes cm^{-2}. This can be reduced to atmospheres, with ample accuracy for our purposes, by multiplication by 10^{-6}.

It is convenient in numerical calculations, following the physicists' custom, to express the χ's in volts. If χ is to be in volts, so must kT, and we find, since $e = 4{\cdot}774 \times 10^{-10}$ E.S.U.,

$$kT = \frac{1{\cdot}37 \times 10^{-16} \times 300}{4{\cdot}774 \times 10^{-10}} T = 8{\cdot}60 \times 10^{-5} T \text{ volts.}$$

Finally, to convert the wave numbers of optical terms into the energies in volts of the corresponding stationary states we have to multiply them by $1{\cdot}234 \times 10^{-4}$.

§ 3. *Numerical Applications.*—We now apply the theory to all optical series for which we have been able to find the necessary stellar data. We quote all lines under Fowler's* notation, and calculate potentials from his wave numbers. For each series considered we first give the lines used and their formulæ, the wave numbers and potentials required, the effective value of $b(T)$ near the maximum, and the value of σ. We then summarise very shortly the evidence for the observed T_{max}, the temperature of maximum development, and give the calculated value of P_e for this and some neighbouring temperatures.

The order is merely that of convenience.

(i.) *H. Balmer Series.* $\lambda 6563 (N/2^2 - N/3^2)$, etc. $N/1^2 = 109678$, $\chi_1 = 13{\cdot}54$; $N/2^2 = 27420$, $\chi_2 = 3{\cdot}385$: $b(T) = 2$, $\sigma = 1$.—The Balmer lines reach their maximum in stars of type A0 (definition of this type),†

* A. Fowler, "Report on Series in Line Spectra," *Phys. Soc. London* (1922).

† *Harv. Annals*, 91 (*H.D. Cat.*), p. 7 (1918).

at a temperature of about 10,000°. For three alternative temperatures we find, using (5),

$$T_{max} = 10{,}000^\circ, \qquad P_e = 1{\cdot}31 \times 10^{-4} \text{ atmos.}$$
$$T_{max} = 11{,}000^\circ, \qquad P_e = 7{\cdot}20 \times 10^{-4} \text{ atmos.}$$
$$T_{max} = 12{,}000^\circ, \qquad P_e = 3{\cdot}07 \times 10^{-3} \text{ atmos.}$$

(ii.) *Mg.*† *Fundamental Series.* $\lambda 4481 (2\delta - 3\phi)$. $1\sigma = 121267$, $\chi_1' = 15{\cdot}00$; $2\delta = 49777$, $\chi'_{2\delta} = 6{\cdot}15$;* $b'(T) = 1$, $\sigma' = 1$.—The line $\lambda 4481$ has its maximum intensity in stars like Sirius,† of type A0, and so effectively $\lambda 4481$ and the Balmer lines have simultaneous maxima. We find

$$T_{max} = 10{,}000^\circ, \qquad P_e = 0{\cdot}82 \times 10^{-4} \text{ atmos.}$$
$$T_{max} = 10{,}220^\circ, \qquad P_e = 1{\cdot}30 \times 10^{-4} \text{ atmos.}$$
$$T_{max} = 11{,}000^\circ, \qquad P_e = 5{\cdot}24 \times 10^{-4} \text{ atmos.}$$
$$T_{max} = 12{,}000^\circ, \qquad P_e = 1{\cdot}08 \times 10^{-3} \text{ atmos.}$$

There is thus theoretically a slight difference in the positions of the maxima for the same value of P_e, but the smallness of the difference seems to afford considerable justification for the detailed theory. If an approximate theory is used, taking account only of the 10 per cent. difference of ionisation potential and ignoring the differing values of χ_2 and $b(T)$, the maximum of $\lambda 4481$ comes out about 1000° higher than that of the Balmer lines—in definite discordance with observation.

(iii.) *Mg. Sharp and Diffuse Singlets.* $\lambda 5711 (1P - 3S)$, $\lambda 5528$ $(1P - 3D)$, $\lambda 4703 (1P - 4D)$, $\lambda 4352 (1P - 5D)$. $1S = 61672$, $\chi_1 = 7{\cdot}61$; $1P = 26621$, $\chi_{1P} = 3{\cdot}285$; $b(T) = 1$, $\sigma = 2$.—There is no perceptible change in the development of these lines between the spectra of sun and spot.‡ The line $\lambda 4703$ is weakened in passing from the sun to α Orionis (Ma), and also in passing from Arcturus (K0) to α Orionis.§ The Harvard sequence of intensities‖ for $\lambda 4352$ gives an increase in intensity along the sequence β Persei (B8), Sirius (A0), α Gem. (A0), Procyon (F5), thereafter remaining constant; that for $\lambda 4703$ gives a maximum about F5 – G0. It is clear that the lines have a maximum, which may perhaps be placed between the sun and the spot, say at a temperature of 5,500°. We find

$$T_{max} = 5{,}000^\circ, \qquad P_e = 2{\cdot}43 \times 10^{-5} \text{ atmos.}$$
$$T_{max} = 5{,}500^\circ, \qquad P_e = 1{\cdot}58 \times 10^{-4} \text{ atmos.}$$
$$T_{max} = 6{,}000^\circ, \qquad P_e = 7{\cdot}76 \times 10^{-4} \text{ atmos.}$$

* The notation $\chi'_{2\delta}$ means the negative energy (in volts) of the 2δ-level of the ionised atom. Similarly $\chi_{1\pi_1}$ means the energy of the $1\pi_1$-level of the neutral atom, and so on. These are the χ_r's of the general theory.

† Lockyer, "Catalogue of 470 of the Brighter Stars" (*Solar Physics Committee Publications*), 1902, plate ii.; Lockyer, *Proc. Roy. Soc.*, A, 84, 429 (1910). See also *H.A.*, 91, 7 (1918), and *H.A.*, 28, pt. i., p. 66 (1897). Some authorities consider the line $\lambda 4481$ Mg^+ to reach its maximum in stars of type A2.

‡ Russell, *Astrophys. Journ.*, 55, 130 (1922). Adams, *Astrophys. Journ*, 30, 105 (1909).

§ Baxandall, *Solar Phys. Pub.* (1914), pp. 20, 31.

‖ *H.A.*, 28, pt. i., pp. 65, 67 (1897).

(iv.) *Mg. Sharp Triplets.* λλ5184, 5173, 5167 ($1p_1$, $1p_2$, $1p_3 - 1s$). $1S = 61672$, $\chi_1 = 7{\cdot}61$; $1p = 39800$, $\chi_{1p} = 4{\cdot}92$; $b(T) = 1$, $\sigma = 2$.—The triplets of the alkaline earths correspond to higher levels than the 1S level of the normal atom, even the principal series of triplets, and so are essentially subordinate series from the present point of view, and should show maxima. The triplet mentioned forms part of the Fraunhofer *b* group, and is best regarded as the first member of the sharp series. It shows little or no change between sun and spot.* The Harvard sequences† indicate a maximum intensity for Arcturus (Ko) We find

$$T_{max} = 5{,}000^\circ, \qquad P_e = 5{\cdot}34 \times 10^{-5} \text{ atmos.}$$
$$T_{max} = 5{,}500^\circ, \qquad P_e = 3{\cdot}45 \times 10^{-4} \text{ atmos.}$$
$$T_{max} = 6{,}000^\circ, \qquad P_e = 1{\cdot}69 \times 10^{-3} \text{ atmos.}$$

For given T_{max}, P_e comes out higher than for the singlets, owing to the smaller $\chi_1 - \chi_2$; hence for given P_e, T_{max} is lower. The observations also suggest a lower T_{max} for this triplet than for the diffuse singlets.

It should be mentioned here that the resonance line $\lambda 4571 (1S - 1p_2)$ definitely increases in intensity from sun to spot. This is in accordance with theory, since it arises from the normal or 1S level.

(v.) *Na. Sharp and Diffuse Doublets.* λλ5688,83($1\pi - 3\delta$), λλ4983,79($1\pi - 4\delta$), λλ4669,65($1\pi - 5\delta$), λλ4498,94($1\pi - 6\delta$). $1\sigma = 41449$, $\chi_1 = 5{\cdot}12$; $1\pi_1 = 24{,}476$, $\chi_{1\pi_1} = 3{\cdot}03$; $b(T) = 1$, $\sigma = 1$.—These lines are much stronger in the spot than in the sun,‡ and become progressively stronger down to Ko stars, perhaps to α Orionis.§ T_{max} is not well determined from this evidence, but it must be less than the sunspot temperature (4500°), *i.e.* about 4000°. We find

$$T_{max} = 3{,}500^\circ, \qquad P_e = 1{\cdot}80 \times 10^{-5} \text{ atmos.}$$
$$T_{max} = 4{,}000^\circ, \qquad P_e = 2{\cdot}17 \times 10^{-4} \text{ atmos.}$$
$$T_{max} = 4{,}500^\circ, \qquad P_e = 1{\cdot}53 \times 10^{-3} \text{ atmos.}$$

The D lines belong to the principal series of the neutral atom, and are of little importance here.

(vi.) *Sr*+. *Principal Doublets.* λλ4216, 4078($1\sigma - 1\pi_1, 1\pi_2$), $1S = 45936$, $\chi_1 = 5{\cdot}665$; $1\sigma = 88953$, $\chi_1' = 10{\cdot}97$; $b(T) = b'(T) = 1$; $\sigma = 2$, $\sigma' = 1$.—These important lines have the same intensity in sun and spot,‖ and reach a maximum in the stellar sequence at perhaps G or K.¶ They increase with increasing luminosity (in the same spectral type) for all low temperature stars; even for giant M's. This should be ascribed to the diminished pressures in the atmospheres of high luminosity stars, which

* Russell, *loc. cit.* Adams, *loc. cit.*

† *H.A.*, *loc. cit.*, p. 74.

‡ Russell, *loc. cit.*

§ *H.A.*, *loc. cit.* Our identification of some of these lines in the Harvard list may be a little uncertain.

‖ Russell, *loc. cit.*

¶ *H.A.*, *loc. cit.*, pp. 61, 63. The evidence is somewhat slight.

will *increase* the concentration of suitable atoms (normal Sr^+) provided they are at a temperature below T_{max}. We find

$$T_{max} = 5{,}500°, \qquad P_e = 2{\cdot}62 \times 10^{-5} \text{ atmos.}$$
$$T_{max} = 6{,}000°, \qquad P_e = 1{\cdot}43 \times 10^{-4} \text{ atmos.}$$

These values are not inconsistent with the available evidence.

It may be mentioned here that the principal singlet λ4607 Sr(1S – 1P) shows a marked increase of intensity in faint dwarfs.* This is consistent with theory, for the concentration of normal neutral Sr atoms will be increased by any decrease of temperature or increase of pressure.

(vii.) *Ba+. Principal Doublets.* λλ4934, 4554($1\sigma - 1\pi_1$, $1\pi_2$). $1S = 42029$, $\chi_1 = 5{\cdot}18$; $1\sigma = 80665$, $\chi_1' = 9{\cdot}95$; $b(T) = b'(T) = 1$; $\sigma = 2$, $\sigma' = 1$.—The pair on the whole is a little stronger in the spot than in the sun.† Baxandall ‡ gives λ4554 as slightly strengthened from Arcturus to α Orionis, strengthened from the sun to α Orionis. We find

$$T_{max} = 5{,}000°, \qquad P_e = 2{\cdot}52 \times 10^{-5} \text{ atmos.}$$
$$T_{max} = 5{,}500°, \qquad P_e = 1{\cdot}58 \times 10^{-4} \text{ atmos.}$$

These temperatures are definitely higher than the T_{max} indicated by the evidence, which is perhaps 4000° or lower. The same is true to a smaller extent for Sr^+. Thus smaller pressures than 10^{-4} atmos. seem to be indicated. It must be remembered, however, that the maximum of the principal series of an ionised atom can never be well marked.

(viii.) *Ca. Sharp and Diffuse Singlets.* λ5513(1P – 3S), λ4847 (1P – 4S), λ7326(1P – 2D), λ5189(1P – 3D). $1S = 49305$, $\chi_1 = 6{\cdot}08$; $1P = 25652$, $\chi_{1P} = 3{\cdot}16$; $b(T) = 1$, $\sigma = 2$.—These lines are all strengthened from the sun to the spot, but we have been unable to trace their behaviour in stellar spectra. The maximum would appear to occur at a temperature below that of the spot (4500°). We find

$$T_{max} = 4{,}000°, \qquad P_e = 2{\cdot}01 \times 10^{-5} \text{ atmos.}$$
$$T_{max} = 4{,}500°, \qquad P_e = 1{\cdot}83 \times 10^{-4} \text{ atmos.}$$
$$T_{max} = 5{,}000°, \qquad P_e = 1{\cdot}16 \times 10^{-3} \text{ atmos.}$$

(ix.) *Ca. Diffuse Triplets.* λλ4455, 4435, 4425($1p_1$, $1p_2$, $1p_3 - 2d$). $1S = 49305$, $\chi_1 = 6{\cdot}08$, $1p_1 = 33989$, $\chi_{1p_1} = 4{\cdot}20$; $b(T) = 1$, $\sigma = 2$.—The line λ4455 shows no variation with absolute magnitude in F-type stars. § It is stronger in the K and M types than in the hotter stars,‖ and reaches very great intensity in dwarf M stars. § It is stronger in the spot than in the sun.¶ T_{max} should therefore certainly be less than the temperature of the spot, say 4000°. This is fairly consistent with the following calculated values. But the great development of λ4455 in dwarf M stars

* Adams and Joy, *Astrophys. Journ.*, **46**, 313 (1917).

† Russell, *loc. cit.*

‡ Baxandall, *loc. cit.* He also gives λ4934, blended with λ4933 Fe, as much strengthened from Arcturus to α Orionis, and as strengthened from the sun to α Orionis. The position of the maximum in the Harvard sequences is obscure.

§ Adams and Joy, *loc. cit.* ‖ *H.A.*, *loc. cit.*, p. 66.

¶ Russell, *loc. cit.*

is not accounted for by the theory, unless in these stars it is mainly a high-level line. We find

$$T_{max} = 3{,}500^\circ, \qquad P_e = 2{\cdot}15 \times 10^{-6} \text{ atmos.}$$
$$T_{max} = 4{,}000^\circ, \qquad P_e = 3{\cdot}78 \times 10^{-5} \text{ atmos.}$$
$$T_{max} = 4{,}500^\circ, \qquad P_e = 3{\cdot}73 \times 10^{-4} \text{ atmos.}$$
$$T_{max} = 5{,}000^\circ, \qquad P_e = 2{\cdot}38 \times 10^{-3} \text{ atmos.}$$

It is interesting to compare the change in maxima with change of series for the same element. As between Ca singlets and Ca triplets, there is a difference of a factor of nearly 2 in the pressure, or of nearly 200° in the temperature.

(x.) *Ca*+. *Principal Doublets.* λλ3968, 3933($1\sigma - 1\pi_1$, $1\pi_2$). $1S = 49305$, $\chi_1 = 6{\cdot}08$; $1\sigma = 95740$, $\chi_1' = 11{\cdot}82$; $b(T) = b'(T) = 1$; $\sigma = 2$, $\sigma' = 1$.—These lines, the H and K lines, reach their greatest intensity * in stars of type Ko, but are very conspicuous also in the stars on either side of types Go and K5. The persistence of a strong intensity through a considerable range of temperature is in conformity with the theory, these lines belonging to the principal series of an ionised atom. We find

$$T_{max} = 5{,}000^\circ, \qquad P_e = 9{\cdot}89 \times 10^{-7} \text{ atmos.}$$
$$T_{max} = 6{,}000^\circ, \qquad P_e = 5{\cdot}02 \times 10^{-5} \text{ atmos.}$$

We encounter here lower pressures than those found hitherto. But the H and K lines are notoriously high-level lines, probably originating in regions of lower pressure than most other lines.

(xi.) *Zn. Sharp Triplets.* λλ4810, 4722, 4680($1p_1$, $1p_2$, $1p_3 - 1s$). $1S = 75767$, $\chi_1 = 9{\cdot}35$; $1p_2 = 43265$, $\chi_{1p_2} = 5{\cdot}35$; $b(T) = 1$, $\sigma = 2$.—This triplet is stronger in the sun than in the spot.† Adams and Joy ‡ have recently found, from an examination of high-dispersion spectra, that compared with the neighbouring arc lines of Fe the lines of the Zn triplet show their greatest relative intensity in Sirius (Ao), are prominent in Procyon (F5), and become somewhat less strong in Arcturus (Ko) than in the solar spectrum. It is unlikely that these lines are stronger in types earlier than Ao, and it is reasonable to assume that the maximum occurs at a temperature somewhat lower than that of Sirius. We find

$$T_{max} = 8{,}000^\circ, \qquad P_e = 8{\cdot}44 \times 10^{-3} \text{ atmos.}$$
$$T_{max} = 9{,}000^\circ, \qquad P_e = 5{\cdot}22 \times 10^{-2} \text{ atmos.}$$
$$T_{max} = 10{,}000^\circ, \qquad P_e = 2{\cdot}37 \times 10^{-1} \text{ atmos.}$$

These are *higher* pressures than those so far found. But it must be observed that these are low-level lines in the sun, § considerably lower, for example, than Ca, Mg, Sr, Ba. The high pressures are at once accounted for if Zn lines are low-level lines in general.

* *H.A.*, 91 (*H.D. Cat.*), p. 8 (1918). † Russell, *loc. cit.*
‡ *Pub. Ast. Soc. Pac.*, 34, 177 (1922).
§ St. John, *Astrophys. Journ.*, 38, 345 (1913); Mitchell, *Astrophys. Journ.*, 38, 407 (1913).

(xii.) *Zn. Diffuse Singlets.* λ6362 (1P – 2D). 1S = 75767, $\chi_1 = 9{\cdot}35$; 1P = 29022, $\chi_{1P} = 3{\cdot}58$; $b(T) = 1$, $\sigma = 2$.—This should have its maximum at a somewhat higher temperature or lower pressure than the sharp triplets. However,* the singlet is less weakened than the triplet in the spot as compared with the sun, which is not in accordance with theory.

(xiii.) *He. Diffuse Doublets.* λ5876 (1π – 2δ), λ4471 (1π – 3δ), λ4026 (1π – 4δ), etc. $\chi_1 = 24{\cdot}7$;† $1\pi_1 = 29224$, $\chi_{1\pi_1} = 3{\cdot}61$; $b(T) = 1$, $\sigma = 2$.—The observed maximum of the general body of He lines ‡ occurs in stars of type B2. We find

$T_{max} = 14{,}000°$, $P_e = 5{\cdot}93 \times 10^{-6}$ atmos.
$T_{max} = 16{,}000°$, $P_e = 1{\cdot}16 \times 10^{-4}$ atmos.
$T_{max} = 17{,}000°$, $P_e = 3{\cdot}97 \times 10^{-4}$ atmos.

The maxima of the diffuse singlets should occur for much the same temperatures and pressures, the 1P term (1P = 27175) being close to the 1π term.

(xiv.) *He*+. "4686 *Series.*" λ4686($4N/3^2 - 4N/4^2$). $4N/1^2 = 438893$, $\chi_1' = 54{\cdot}2$; $4N/3^2 = 48766$, $\chi_3' = 6{\cdot}02$; $b'(T) = 2$, $\sigma' = 1$.—The line λ4686 is increasing in intensity with increasing temperature amongst the stars of type O, and there is no observed maximum. The position is complicated owing to its tendency to appear bright, and, as already remarked, the present method is incapable of dealing with emission lines. We find

$T_{max} = 30{,}000°$, $P_e = 5{\cdot}1 \times 10^{-6}$ atmos.
$T_{max} = 35{,}000°$, $P_e = 1{\cdot}07 \times 10^{-4}$ atmos.

(xv.) *He*+. *Pickering Series.* λ5412($4N/4^2 - 4N/7^2$), etc. $4N/1^2 = 438893$, $\chi_1' = 54{\cdot}2$; $4N/4^2 = 27431$, $\chi_4' = 3{\cdot}39$; $b'(T) = 2$, $\sigma' = 1$.—This series also is increasing in intensity. In particular it is increasing in intensity through the sequence of O-type stars considered by H. H. Plaskett.§ We find

$T_{max} = 30{,}000°$, $P_e = 3{\cdot}80 \times 10^{-6}$ atmos.
$T_{max} = 33{,}000°$, $P_e = 3{\cdot}22 \times 10^{-5}$ atmos.
$T_{max} = 35{,}000°$, $P_e = 1{\cdot}18 \times 10^{-4}$ atmos.

We defer comment on these values to § 5 below.

§ 4. *General Evidence on Pressures in Reversing Layers.*—The applications of the theory of maximum intensities which we have developed lead unmistakably to a normal reversing layer pressure of the order of 10^{-4} atmosphere for most of the elements it has been possible to consider, with suggestions of higher and lower pressures for lines which are known to be low-level and high-level lines in the sun. It must be

* Russell, *loc. cit.*

† This, *not* 25·4 volts is about the correct ionisation potential. It is based on Lyman's observation of the true principal series of singlets, *Nature*, **110**, 278 (1922). His 0S-term gives $\chi_1 = 24{\cdot}5$ volts: we have used 24·7 in error. See also Franck, *Zeit. für Physik*, **11**, 155 (1922); Davies, *Phil. Mag.*, **45**, 786 (1923). The term called by A. Fowler 1S is properly the 2S term.

‡ *H.A.*, **91** (*H.D. Cat.*), p. 6 (1918). § *Loc. cit.*, p. 356.

remembered that since we have been dealing with maxima, the pressures we have found are those for the levels corresponding to the formation of *intense* lines. We propose briefly to review the existing evidence from all sources as to reversing layer pressures.

(*a*) *Pressure Shifts of Spectral Lines.*—Early comparisons of wave-length between arc lines and solar lines yielded pressures in the sun of several atmospheres. Thus Jewell, Mohler, and Humphreys,* from a study of various metallic lines, inferred pressures from 2 to 7 atmospheres; and fourteen years later Fabry and Buisson,† from a detailed study of the iron arc, concluded that the iron vapour in the sun responsible for the absorption lines was at a pressure of 5 to 6 atmospheres. But, re-discussing ‡ their observations in 1921, they state that the differences between the sun and the iron arc *in vacuo* can be thoroughly explained assuming a negligible pressure in the reversing layer, but taking account of the Einstein effect. The same conclusion is reached by Perot§ from measures of cyanogen bands and Mg lines. His last quoted paper is particularly striking. Using the fact that the ratio of the wave-lengths of similar lines possessing different pressure coefficients is unaltered by the Doppler or Einstein effects but is altered by pressure, he finds that the pressure in the solar reversing layer contributing the Mg *b* triplet is negligibly small. No such investigation can distinguish between pressures of, say, 0 and 10^{-1} atmosphere. For Fe lines Perot‖ has recently found a pressure of 30 cms. of mercury, and Salet¶ has found very small pressures. Deductions of pressures by these methods are now known to be exceptionally difficult and must still be regarded with caution, and the results of the investigations known to be in progress at Mount Wilson will be awaited with interest. But it is clear that there is no longer any opposition from evidence of this type to pressures of the order of 10^{-4} atmosphere.

(*b*) *Dissociation and Ionisation.*—Another line of attack is by the use of dissociation or ionisation theories. The case of TiO_2 has been discussed by Atkinson.** The band spectrum of this compound—a leading feature in stars of type M, existing also in K5 stars—disappears with increasing temperature at K0 stars. Atkinson showed that this indicated a reversing layer pressure between 10^{-3} and 10^{-7} atmosphere, with 10^{-5} or 10^{-6} atmosphere as a fairly probable value—the argument being that higher pressures than these should so retard dissociation of the molecule with increasing temperature as to permit a development of the flutings in K0 stars. Saha's results on disappearance points already tend in the same direction, though they have not yet been presented systematically. But St. John, in an address to the British Astronomical Association in May 1922, announced that a tentative discussion on these lines had led him to the view that the pressure of the solar reversing layer was about 10^{-2} or 10^{-3} atmosphere. Again, in a footnote in his

* *Astrophys. Journ.*, **3**, 138 (1896).
† *Astrophys. Journ.*, **31**, 113 (1910). Also *C.R.*, **148**, 688 (1909).
‡ *C.R.*, **172**, 1020 (1921).
§ *C.R.*, 170, 988; **171**, 229 (1920); **172**, 578 (1921); **174**, 151 (1922). *Journ. de Phys.*, ser. 6, **3**, 101 (1922).
‖ *C.R.*, **174**, 933 (1922). ¶ *C.R.*, **174**, 151 (1922).
** *Monthly Notices*, **82**, 396 (1922).

memoir * on "Three O-Type Stars," H. H. Plaskett mentions that recent unpublished investigations of H. N. Russell, the nature of which is not stated, indicate a pressure of 10^{-3} or 10^{-4} atmosphere.

(*c*) *Equilibrium of the Solar Atmosphere.*—There are various difficulties connected with the equilibrium of the solar chromosphere and reversing layer, and though this is not the occasion to enter into them deeply, it will be shown that consideration of them leads to the view that the pressures in the reversing layer are likely to be small. It is well known that purely gravitational equilibrium would require a very sharply bounded sun, with no appreciable chromosphere, and consequently radiation-pressure is invoked, usually in a vague way, to account for the existence of the chromosphere. Now Eddington † has pointed out that on the sun radiation-pressure is capable of supporting only an extremely small mass per unit area; making assumptions which were in fact far too liberal, he found that the utmost permissible mass is a milligram per cm^2. But it is not usually realised that even so small a mass as this cannot be supported by radiation-pressure unless the coefficient of absorption of the gas is extremely large. The argument is briefly that, owing to the large value of g, for equilibrium a small mass must absorb a large amount of radiation, hence the large mass-absorption coefficient.

Let us consider the high-level Ca^+ vapour. It may be objected that St. John's work ‡ shows that the Ca^+ vapour is not in equilibrium, since he found velocities of ascent and descent of 2 kms. and 1 km. per sec. respectively. However, these are of the order of magnitude of the velocity of thermal agitation of Ca atoms at 5000° (namely, 1·76 kms. per sec.), and the continued existence of oppositely directed convection currents of this magnitude is almost incomprehensible. It would appear to be sufficient, at present, to account for mere equilibrium. Consider for definiteness an atom absorbing the K line λ3933 Ca^+. The momentum associated with one quantum § of light of this frequency is $h\nu/c = h/\lambda = 1{\cdot}67 \times 10^{-22}$. The mass of the Ca atom is $40 \times 1{\cdot}64 \times 10^{-24}$ grams, so that for each quantum absorbed the Ca atom receives a velocity increment of 2·54 cm. sec.$^{-1}$. But on the sun $g = 2{\cdot}7 \times 10^4$ cm. sec.$^{-2}$—roughly equivalent to a velocity increment of a quarter of a kilometre per second every second. Hence for a single atom to be maintained in equilibrium under the influence of radiation-pressure only (assuming continual re-emission of the absorbed energy in random directions), it must absorb and re-emit $1{\cdot}06 \times 10^4$ quanta per second. From the point of view of atomic structure there does not seem anything impossible in this figure,‖ since the time of reorganisa-

* *Pub. Domin. Astrophys. Obs.*, 1, No. 30, 377 (1922).

† *Monthly Notices*, 80, 723 (1920).

‡ *Astrophys. Journ.*, 32, 36 (1910).

§ The bearing of the quantum theory of radiation-pressure on solar and stellar phenomena has been previously considered by Saha, *Astrophys. Journ.*, 50, 220 (1919); *Nature*, 107, 488 (1921). He drew attention to the fact that the action of radiation-pressure on a single atom is necessarily both impulsive and selective, and that its magnitude depends simply on the number of quanta the atom can absorb and re-radiate per second.

‖ Foote and Mohler, *The Origin of Spectra* (1922), p. 98, quote observations by Zahn, *Verh. d. Phys. Ges.*, 15, 1203 (1913), which show that in a sodium flame each atom emits 2000 quanta of D-radiation per second.

tion * of an atom is supposed to be of the order of 10^{-8} sec., but it implies a very large mass-absorption coefficient.

To calculate the latter we must make some estimate of the amount of radiation available. Taking the K line to be 10 A.U. wide † and taking 6000° for the solar temperature, the amount of radiation $I_\lambda d\lambda$ per unit solid angle per sq. cm. per sec. is from Planck's formula about 3×10^7 ergs. This is equivalent to about 6×10^{18} quanta. Adding an equal amount for the H line, the number of quanta per second is about $1{\cdot}2 \times 10^{19}$. Taking account of the spread of this radiation over the solid angle 2π, it is found that if the radiation is all absorbed the mass per unit area which can be sustained by radiation-pressure is

$$\frac{1}{\pi} \cdot \frac{1{\cdot}2 \times 10^{19}}{1{\cdot}06 \times 10^4} \times 40 \times 1{\cdot}64 \times 10^{-24} = 2{\cdot}4 \times 10^{-8} \text{ gram cm.}^{-2}.$$

If then the whole of the radiation is absorbed, the order of magnitude of the mass-absorption coefficient k is given by $kdm = 1$, so that k is of the order of magnitude of $0{\cdot}4 \times 10^8$. This can, of course, be calculated from the macroscopic magnitudes without bringing in the quantum theory. It seems impossible to avoid accepting so high a value ‡ if equilibrium is governed by radiation-pressure. Actually it must be still larger; for since we have assumed random re-emission, therefore half the amount absorbed must be radiated forwards, so that our assumptions imply an intensity in the H and K lines equal to half that in the neighbouring continuous spectrum. But the pressure of radiation is proportional to the net flux of energy, in this case the intensity in the lines, and hence the available energy may be of the order of 5 or 10 times smaller than we have supposed.

The vapour of Ca^+ has been observed § to extend to at least 14,000 kms. If the mass $2{\cdot}4 \times 10^{-8}$ gram cm.$^{-2}$ occupies a column of, say, 5000 kms. length at 5000°, its mean pressure will be about 5×10^{-13} atmos. The average density is equivalent to about 7×10^5 atoms per cm.^{3}, and the number of collisions per atom per second is about 2×10^{-4}. Practically, then, there are no collisions, and each atom is as it were suspended in equilibrium under the average upward pressure of the quantum impulses.

The view of the chromosphere just put forward may be contrasted with that of Saha,|| who suggested that the pressure falls to 10^{-4} atmos. at 5000 kms.; this is Mitchell's level for the termination of the line $\lambda 4227$ Ca ($1S - 1P$), and ionisation of Ca is according to Saha "complete"

* See, *e.g.*, Foote and Mohler, *loc. cit.*, p. 96.

† The width of the portion of the K line originating in the chromosphere, which alone is here relevant, is much smaller than this.

‡ Foote and Mohler, *loc. cit.*, p. 89, state that a layer of mercury vapour 0·5 mm. thick at a pressure of 0·001 mm. may reduce the intensity of a beam of resonance radiation $\lambda 2537 (1S - 2p_2)$ to $\frac{1}{3}$ of its initial value. From this it appears that the mass-absorption coefficient is about 2×10^6. The H and K lines would be the resonance lines of Ca^+, and the large value found in the text is therefore plausible.

§ Mitchell, *Astrophys. Journ.*, **38**, 407 (1913).

|| *Phil. Mag.*, **40**, 481 (1920).

at 10^{-4} atmos. The difficulty again arises that we do not know how few un-ionised atoms need remain for the line still to appear.

It will have been noticed that in attributing equilibrium solely to radiation-pressure we have ignored any support due to a pressure gradient. We now see that this is justified, since the pressures are so small and collisions so rare.

It does not appear to be possible to infer the pressure in the reversing layer directly from the chromospheric pressure, but the following considerations would seem to be of some help. If the solar reversing layer be supposed to be a gas of mean molecular weight 40, and if it were in purely gravitational equilibrium at a temperature of 6000°, then, using the formula $p/p_0 = e^{-mgx/RT}$, it is found that the pressure would be multiplied by a factor of $10^{9\cdot4}$ every 100 kms. of descent into the sun's interior. Now the observations of St. John* and others on the levels of lines show that these vary by some hundreds of kilometres (apart from high-level chromospheric lines). The "level" of a line must be regarded as the mean depth to which we see into the sun in the particular wave-length. It is almost inconceivable that we could thus see into depths differing in total pressure by a factor of 10^{19} or 10^{28} (corresponding to 200 kms. and 300 kms.), even paying attention to the circumstance that it is only the partial pressure of an element which is significant. The pressure gradients in the reversing layer must then be small. We infer that the reversing layer is not in purely gravitational equilibrium, and it is natural to suppose that it is precisely the region of transition in which equilibrium under an appreciable pressure gradient † gives place to low-pressure equilibrium under radiation-pressure. But the masses that can be supported, per unit area, are still very small—a mean pressure of an atmosphere continued for 100 kms. would mean a far larger mass than radiation-pressure could possibly support. Moreover, entirely different lines of evidence ‡ have suggested that molecular scattering of photospheric light by the solar atmosphere is negligible, which again would be incompatible with reversing layer pressures of the order of an atmosphere. To attribute a pressure of 10^{-4} atmos. to a particular level in the reversing layer—about 10^9 times the pressure we have calculated for the chromosphere—is therefore to suppose that the transition is equivalent to about 100 kms. of Ca vapour in gravitational equilibrium.

No particular weight should be attached to the precise numerical magnitudes we have been adducing. We cannot even show definitely that a pressure so large as 10^{-1} or 10^0 atmos. for a particular level in the reversing layer is impossible. But the general trend of the argument is to suggest that very low pressures for the reversing layer are highly probable; indeed the discussion points in the direction of pressures

* *Astrophys. Journ.*, **38**, 341 (1913).

† With Eddington's value of the mass-absorption coefficient for the *interior* of a star (of the order 10 or 10^2), the ratio of radiation-pressure to gravity near the surface of the sun is negligible. It must be remembered that the value derived by Eddington applies to the integrated radiation of all wave-lengths, whilst we have been discussing the values for monochromatic radiation.

‡ Pannekoek, *B.A.N.*, No. 19, p. 113 (1922); Milne, *Phil. Trans.*, A, 223, 225 (1922).

lower than 10^{-4} atmos. rather than of higher pressures. Further, the demonstration that the gas in the chromosphere is behaving as though it possesses a large mass-absorption coefficient removes the difficulty which might be felt as to whether a reversing layer at so low a pressure could contain enough matter to yield lines of the observed intensity.

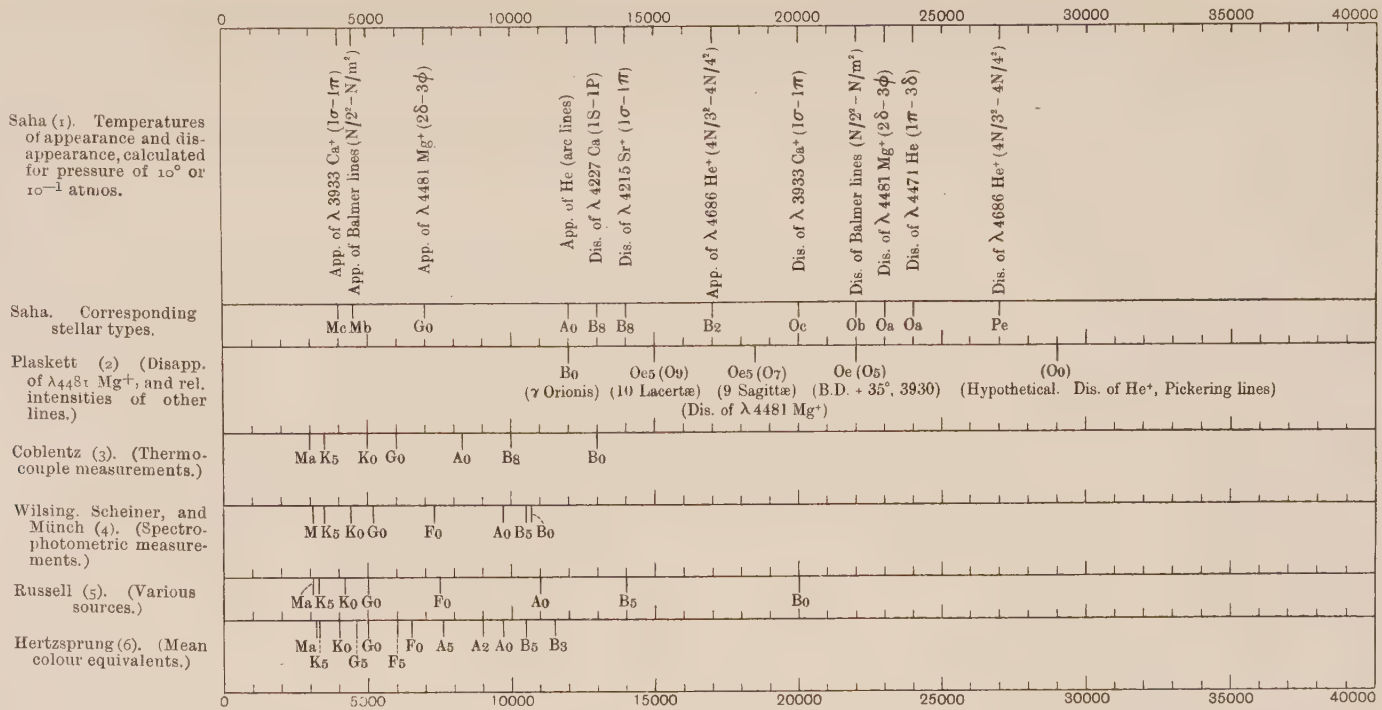
To summarise, the general tendency of recent investigations has been to attribute steadily lower pressures to reversing layers, and the evidence is perfectly compatible with the low values of the pressure found in this paper.

§ 5. *The Stellar Temperature Scale.*—We now assume for definiteness that throughout the stellar sequence the partial electron pressure P_e in a reversing layer which is the seat of a strong absorption line is $1{\cdot}31 \times 10^{-4}$ atmos. This is the value we found to give a maximum for the Balmer series at 10,000°. The following table shows the calculated temperatures of maxima for $P_e = 1{\cdot}31 \times 10^{-4}$ atmos., obtained by logarithmic interpolation amongst the calculated values of § 4. The second column gives the spectral type in which the lines mentioned reach a maximum, so far as we have been able to estimate it from the evidence adduced in § 4.

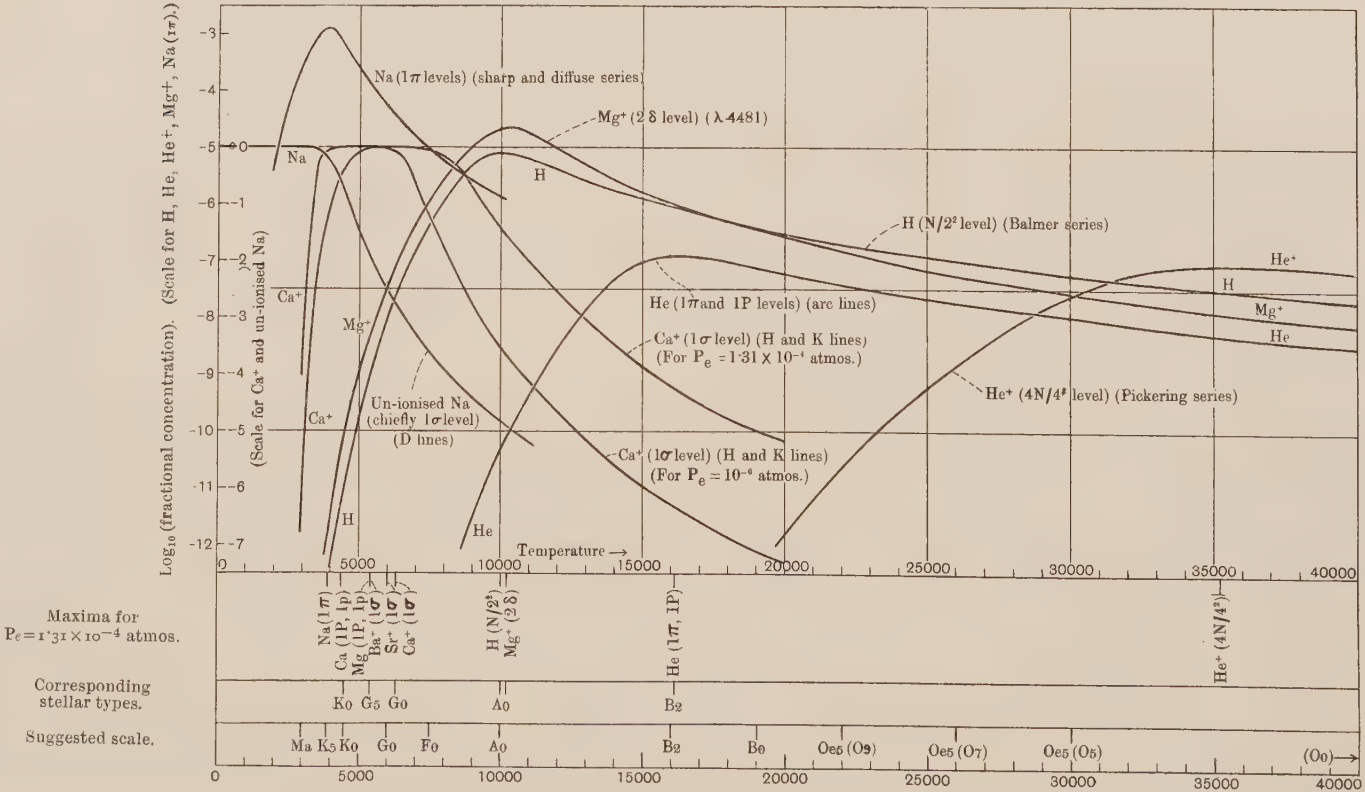
$T_{max.}$	Type.	Maximum of.
3,900°	K5	Na ; $1\pi - m\sigma$, $1\pi - m\delta$.
4,270°	K5	Ca ; $1p - md$.
4,420°	K5	Ca ; $1P - mS$, $1P - mD$.
5,250°	G5	Mg ; $1p - ms$.
5,440°	G5	Mg ; $1P - mS$, $1P - mD$.
*5,450°	Ko – K5	Ba^+ ; $1\sigma - m\pi$.
*5,970°	Go – Ko	Sr^+ ; $1\sigma - m\pi$.
*6,290°	Ko	Ca^+ ; $1\sigma - m\pi$.
10,000°	Ao	H ; Balmer series.
10,220°	Ao	Mg^+ ; $2\delta - m\phi$.
16,100°	B2	He ; arc lines ($1\pi - m\delta$, etc.).
35,200°	...	He^+ ; "4686" and Pickering series.

The general sequence of stellar classes, series maxima, and temperatures is shown diagrammatically in Plates 15, 16, together with temperature scales from other sources for comparison. The curves show the fractional concentrations, plotted against temperature, of atoms in states capable of absorbing the lines mentioned. The curves for subordinate series are calculated from equation (4), § 2 ; those for the principal series of the ionised atom (Ca^+) from equation (9). A curve is also given for the fraction of neutral atoms, in the case of Na, calculated from equation (3) ; except at temperatures which are "high" for the neutral atom considered, the neutral atoms will be almost all in the normal (1σ) state, so that this curve suffices to represent the intensity

* These lines are probably lower pressure lines, with a lower temperature maximum.



(1) Saha, *Proc. Roy. Soc.*, **A**, **99**, 151 (1921).
(2) Plaskett, "The Spectra of Three O-Type Stars," *Pub. Dom. Astrophys. Obs.*, **1**, No. 30 (1922).
(3) Coblentz, *Proc. Nat. Acad. Sci.*, **8**, 49 (1922); *Pop. Astron.*, **31**, No. 2 (1923). (The measurements refer only to a few individual stars.)
(4) Wilsing, Scheiner, and Münch, *Pub. Astrophys. Obs. Potsdam*, **24**, No. 74 (1919).
(5) Russell, *Nature*, **93**, 227 (1914). The lower temperatures appear to be derived chiefly from Wilsing and Scheiner's earlier results, *loc. cit.*, **19**, No. 1 (1909); the higher ones from the mean colour indices of King, Parkhurst, and Schwarzschild, apparently deduced by the method described by Russell, *P.A.S.P.*, **32**, 307 (1920).
(6) Hertzsprung, *Leiden Ann.*, **14**, pt. i. p. 15 (1922). (The mean colour equivalents from all sources, taken according to the Harvard classification, have been converted into temperatures.)



(*Note.*—The partial electron pressure $P_e = 1{\cdot}31 \times 10^{-4}$ is chosen so as to give a maximum for the Balmer lines at $T_{max} = 10{,}000°$. Curves for all elements are then calculated for this pressure. An alternative curve is given for Ca^+.)

of the principal series of the neutral atom. The ordinates of the curves give the logarithms of the fractional concentrations, to base 10 (*i.e.* $\log_{10} n_r$, etc.). For Ca^+ and un-ionised Na the origin of the ordinate-scale has been shifted. The curves plotted illustrate at a glance the differing behaviours of the principal series of the neutral atom, the principal series of the ionised atom, and subordinate series, as described in § 1. To avoid confusion only a few of the curves whose maxima have been worked out are shown.

Below the diagrams we give a suggested scale of stellar temperatures based on the observed maxima. It is purely provisional. It may be desirable, for instance, to assume a temperature of 12,000° for Ao stars, corresponding to $P_e = 3 \times 10^{-3}$ atmos.; this would have the effect of raising all the temperatures. It should be remembered that the assumption of a value of T_{max} for the Balmer lines is the only place where the theory is adjusted to fit the observations; indeed, if a value $P_e = 10^{-4}$ atmos. can be assumed on other grounds, the temperature scale to which we are led is independent of any adjustment.

The spectral type B2 is the earliest to which we have been able to assign a temperature (16,000°) from consideration of maximum intensities, and above this the scale is largely a matter of guesswork.* We have placed the type Bo 30000° above B2, and then used the *temperature steps*, slightly increased, found by Plaskett for his three stars of type O.

We give also on the plates a selection of temperature scales from other authorities. The temperatures obtained by Saha from marginal appearances depend on an assumed pressure of 10^{0} or 10^{-1} atmos., except for the disappearance of λ4686 He^+, for which he took a pressure of 10^{-5} atmos. The scale of Hertzsprung has been deduced from the mean values of c_2/T, which he obtained by combining colour equivalents from all available sources. It should be noted, however, that his method was to take the spectro-photometrically determined scale of Wilsing as a standard; hence his scale, like Wilsing's, gives low temperatures for early-type stars.†

The various scales we quote are in no sense strictly comparable, as they refer to different selections of stars, etc. Moreover, we have ignored throughout the known differences of temperature between giants and

* When the series relations of silicon or carbon are known, the temperature of Bo stars will be obtainable. From the diagram given by Plaskett, *loc. cit.*, p. 356, it appears that λ4089 (Si IV = Si^{+++}) and λ4648 (C^{++}) have maxima in ε Orionis. Similarly from the same diagram the line λ4097(N^+) has a maximum in the star 9 Sagittæ, of type Oe5. It should be possible, too, from more detailed stellar data to make use of the series relations discovered by Catalan for lines of Mn, *Phil. Trans.*, A, 223, 127 (1922).

† Some of the separate scales thus combined yield much higher temperatures for early-type stars, *e.g.* the values of c_2/T obtained by Wilsing by an independent colorimetric method, *Potsdam Pub.*, Bd. 24, No. 76 (1920). The three separate sets of determinations by Wilsing and his colleagues, namely, the one just mentioned (giving very high temperatures for the early-type stars) and the two spectro-photometric ones, *Potsdam Pub.* (1910, 1919), giving low temperatures, have as a mean the following scale, according to Scheiner-Graff, *Astrophysik*, Leipzig (1922), 362:—

Bo.	B5.	Ao.	A5.	Fo.	Go.	Ko.	K5.	Ma.
15,800°	11,900°	9500°	7900°	6700°	5100°	4000°	3600°	3300°

dwarfs (giants appear to be some 10 or 20 per cent. cooler than dwarfs of the same spectral type). Our object is simply to obtain a rough survey of the existing estimates of stellar temperatures, and to see how (if at all) the theory developed in the present paper suggests a modification of them.

Roughly speaking, all the scales are in approximate agreement for types cooler than A0, and our investigation of maxima confirms them. For earlier types our investigation confirms those scales which give the highest temperatures. It agrees best with the scale given by Russell. It can only be lowered by assuming reversing layer pressures lower than $P_e = 10^{-4}$ atmos.

By taking a pressure of 10^{-5} atmos. Saha has calculated the temperature of disappearance of λ4686 He^+ (namely, of the complete second-stage ionisation of He) as $25{,}000^\circ - 30{,}000^\circ$; and Plaskett has given 29,000° as the temperature of a hypothetical class O0, characterised by the disappearance of the Pickering lines. We, on the other hand, find that λ4686 He^+ and the Pickering lines should read a *maximum* at 35,000°. It is interesting to analyse the source of the discrepancy. In part it is due to Saha's smaller pressure, 10^{-5} atmos.; in part it is due to Saha's working only with the electron pressure derived from the ionisation of the element in question; in part it is due to our introduction of $b(1) = 2$. But the main and more interesting reason is that λ4686 He^+ and the Pickering lines reach a maximum only when the second-stage ionisation of He is in fact nearing completion. From formula (6), § 2, it is easily calculated that when the Pickering lines are at their maximum the degree of ionisation is 0·82. The nearness of this to unity is due to the smallness of χ_r compared to χ_1. It is true that at 35,000° the proportion of He^+ atoms remaining is small. But the fraction of these in the state suitable for absorbing the Pickering lines is relatively large, and the two factors work together to give a maximum of the Pickering lines at a comparatively advanced stage of ionisation. This example particularly illustrates some of the features of the present theory.

§ 6. *Discussion.*—In this concluding section it is not our purpose to do more than discuss very briefly one or two outstanding points which arise out of the theory. We notice in the first place that apart from details there is a very satisfactory general agreement between theory and observation of the positions of maxima. The next point of interest is the strikingly small fractional concentration which can yield strongly a line of a subordinate series. For example, we see from the figure that at the maximum intensity of the sharp and diffuse series of Na only one atom in $10^{2\cdot9}$ is effective ($\log_{10} n_r = -2\cdot9$); at that of λ4481 Mg^+, 1 atom in $10^{4\cdot7}$; at that of the Balmer lines, 1 atom in $10^{5\cdot1}$; at that of the He arc lines or of the lines of He^+, 1 atom in 10^7. This at once shows up the danger of making inferences from marginal appearances. Saha has tended to assume that "complete ionisation" means a remaining fraction of less than about 10^{-2} or 10^{-3}. It is clear from our study of subordinate series that for the disappearance of, say, a principal line by complete ionisation the fractional concentration of the normal neutral atoms must be reduced in general at least below 10^{-8}.

A further point is that if we place on the temperature scale the observed marginal appearances of one of the lines for which the distribution curve is drawn, we find that appearance and disappearance with increasing temperature do not seem to occur at the same relative concentration. The arc lines of He appear, for instance, at 10,000° at a relative concentration of suitable atoms of about 10^{-10}. But after the maximum the relative concentration decreases only slowly, and even at 35,000° it is still only a little less than 10^{-8}, so that these lines should be still far from disappearing. According to Saha, λ4471 He disappears in Oa stars to which he attributes a temperature of 24,000°; the difficulty remains if we attribute a temperature 20,000° higher. This is no doubt a defect of the theory. One's impression of the observed decay in the intensity of a line after reaching a maximum is that the rate is not so much slower than its rate of rise as is indicated by the relative shapes of the two sides of the distribution curve.

In this discussion any possible effect of the second and later terms in $b(\mathrm{T})$ has been ignored. A more exact discussion taking into account the change in value of $b(\mathrm{T})$ must lead to slightly steeper slopes on the high temperature side, but it is doubtful if the effect will remove the discrepancy. The fractional concentration of atoms suitable for absorbing a subordinate series has been given as

$$n_r = \frac{q_r e^{\chi_r/k\mathrm{T}}}{b(\mathrm{T})e^{\chi_1/k\mathrm{T}} + a\mathrm{T}^{\frac{5}{2}}},$$

where

$$b(\mathrm{T}) = q_1 + \Sigma q_r e^{-(\chi_1 - \chi_r)/k\mathrm{T}}.$$

Now the number of terms in $b(\mathrm{T})$ for which χ_r is comparable with χ_1 is small. Thus if the sum of the weights of the second and subsequent terms is N, $b(\mathrm{T})$ will exceed but cannot greatly exceed

$$q_1 + \mathrm{N}e^{-\chi_1/k\mathrm{T}}.$$

Near most of the maxima discussed the two terms in the denominator are approximately equal at about $10^{7\cdot5}$. It follows that N must be about 10^6 to affect n_r appreciably near its maximum. At a temperature of $2\cdot5\ \mathrm{T}_{\max}$ the term $a\mathrm{T}^{\frac{5}{2}}$ will become about $10^{8\cdot5}$ and the factor $e^{\chi_1/k\mathrm{T}\chi}$ will fall to 10^3. The extra terms in $b(\mathrm{T})$ may here begin seriously to increase its value; 10^6 terms would increase it from q_1 to at least 10^3, and retain $b(\mathrm{T})e^{\chi_1/k\mathrm{T}}$ at 10^6. But this is still small compared to $a\mathrm{T}^{\frac{5}{2}}$, and scarcely affects n_r. The very large orbits involved in the later terms make it unlikely that as many as or more than 10^6 terms can be effectively present. We must conclude therefore that for the moment the theory is inadequate on this point.

It is interesting to notice that the relative concentration of the Mg^+ atom absorbing λ4481 falls *below* that of H atoms absorbing the Balmer series for high temperatures. This is in the right direction to account for the fact that the Balmer series persists beyond λ4481 Mg^+, but the theoretical effect is hardly adequate. It is probably a question of relative abundance, as suggested by Plaskett. But even here there are

difficulties when the relative concentrations at appearance points are compared.

We reserve for a later communication further points which arise from a study of the curves and in other applications of the theory here given. We conclude by expressing the hope that, if our method commends itself, investigators who have access to extensive collections of stellar spectra will be able to make more exact determinations of the positions of maxima of absorption lines, whose series relations are known, than we have been able to do in this paper.

We wish to express our indebtedness to Messrs. F. E. Baxandall and C. P. Butler of the Solar Physics Observatory, Cambridge, for kindly assisting us with numerous references and other information.

The Proximity of Venus to the Sun during the Total Solar Eclipse of 1923 *September* 10. By J. H. Worthington.

It seems to have hitherto escaped notice that Venus will be extremely near the Sun's edge during the total eclipse of September 10 next. This fact should have more or less interest for astronomers generally, if indeed the proximity might not have some scientific possibilities. According to the American Ephemeris, Venus reaches superior conjunction September 9 days 23 hours.

At the time of geocentric conjunction of Sun and Moon (Sept. $10^d\ 8^h\ 30^m$ G.M.T.) Venus will be about $1^\circ\ 14'$ N. and 2 m. 45 secs. following the Sun. It will therefore appear just over $1\frac{1}{2}^\circ$ from the Sun's centre during totality.

Such an alignment of four celestial bodies—the Earth, Moon, Sun, and Venus is extremely rare, owing to the incommensurability of their orbital periods. I find by referring to Oppolzer that this occurrence will not be repeated this century, although the condition is approached during the annular eclipse of 1957, and the totals of 1962 and 1992.

It seems that Venus shining through the outer corona and an immense section of the zodiacal light on this occasion should offer an opportunity to study the absorption spectra of these solar appendages.

Note on the Reception of Wireless Time-Signals. By W. E. and F. B. Cooke.

In the *Monthly Notices R.A.S.*, **77**, 469, 1917, we suggested a method for the reception of vernier signals, whereby personal equation is eliminated.

That method, with important modifications as hereunder, has been tested on a number of occasions, and has proved highly satisfactory.

It should not be confused with eye-and-ear methods. It is quite automatic. The apparatus picks out and announces each coincidence.

The function of the observer is to record the moment of the first "pip" after a short silence. This is always a whole second, and there can be no variation of tenths or hundredths in the estimate, as is the case with eye-and-ear methods.

The following modification of our original method is now always adopted. It almost entirely eliminates unpleasant telephone clicks. The incoming circuit is not broken by the local clock relay. It passes through a telephone transformer directly into the telephones. The local clock-relay "shorts" the telephones at each beat. The click due to this shorting is barely audible and causes no trouble whatever. It is essential that the duration of relay-closing should exceed that of the wireless signals. The attachment of a little pendulum to the armature of the relay, as explained in our former paper, is quite satisfactory. We are not suggesting that this method is in any way superior to chronographic registration, because we have had no experience with the latter. Our distance from high power stations is too great. Under present circumstances, however, we still think this is the best method for the reception in Australia of vernier signals from Europe or America, and we suggest that it should have a place amongst the experimental work of the Commission de l'heure.